ROBINSON'S
NEW
RUDIMENTS OF ARITHMETIC

AMERICAN BOOK COMPANY
NEW YORK · CINCINNATI · CHICAGO

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ROB'S. NEW RUL'S AR.

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PREFACE.

ROBINSON'S Rudiments of Arithmetic, in its various editions, has probably had a greater sale than any arithmetic of its grade ever published. Although this edition is a thorough revision, all the features which have contributed to the popularity of the Rudiments have been retained, and the changes made have been only such as would bring the book up to date and make it even more valuable to teachers and pupils.

One very valuable feature of the revision consists in the addition of about forty pages of Introductory Exercises. These adapt the book for use in a two-book series, in connection with the Practical Arithmetic. They are of a general character, and are designed especially as suggestions. In the hands of an experienced teacher, they may be greatly extended, and will afford invaluable exercise upon the elementary principles of arithmetic.

On the other hand, if preferred, the book may be used as the second of a three-book series, in which case the Introductory Exercises may be omitted.

Throughout the book, the design has been to present the rudiments of arithmetic in as simple and attractive a form as is possible, consistent with a clear and logical presentation. In pursuance of this object, theory has always been subordinated to practice. The examples are not only of a thoroughly practical character, but are of a nature calculated to help the student to facility and accuracy in the application of the principles of the science.

The book, in its present shape, will commend itself to all by its handsome and clear typography, the logical sequence of the subjects, and its clear, though brief, illustrations and explanations.

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INTRODUCTORY EXERCISES.

LESSON I.

1. How many horses do you see in the picture? How many dogs? How many baskets?
2. Hold up one finger. Hold up one more.
3. One finger and one more are how many fingers?
4. Two fingers are how many more than one finger?
5. One finger is how many less than two fingers?
6. Make one short line on your slate. I
7. One line and one more are how many lines? II
8. How many ones make two?
9. Rub out one line and how many are left?
10. Two lines less one line are how many lines?
11. What is a single thing called?
12. Write the word *one* on your slate. *One.*
13. Make the figure for *one* on your slate. *1.*
14. How many are one and one more? ***Two.***

LESSON II.

1. How many girls are there in this picture? How many birds do you see?
2. How many roses has one girl in her hand?
3. How many roses do you see in the other girl's apron? How many feet has she?
4. One bird and one bird are how many birds?
5. Two are how many more than one?
6. How many ones make two?
7. How many eyes have you? How many ears?
8. Hold up two fingers. Hold up one more.
9. Make two short lines on your slate. II
10. Make one more. How many are there now? III
11. Two lines and one line are how many lines?
12. One and two are how many?
13. Two lines are how many less than three lines?
14. Three lines are how many more than two lines?
15. Rub out one line; how many are left?
16. Rub out two more lines; how many are left?
17. Write the word *two* on your slate. *Two.*
18. Make the figure for *two* on your slate. *2.*
19. How many are two and one more? ***Three.***

LESSON III.

1. How many squirrels are there in the picture? How many bees? How many squirrels with a nut?
2. Two squirrels and one squirrel are how many?
3. Three bees are how many more than 2 bees?
4. Three less one are how many? Three less 2?
5. How many ones make three?
6. How many are 2 and 1? 1 and 2?
7. Make three short lines on your slate. III
8. Rub out one line; how many are left?
9. Rub out one more; how many are left?
10. Three less three are how many?
11. Make three lines again. Now one more. IIII
12. Three lines and one line are how many lines?
13. Hold up one finger. Two fingers. One more. How many fingers are now up?
14. Count three. How many ones are there in three?
15. How many 2's are there in three?

Ans. One 2 and 1 over.

16. Write the word *three* on your slate. *Three.*
17. Make the figure that stands for *three.* *3.*
18. How many are three and one more? ***Four.***

LESSON IV.

1. How many birds are there in the picture? How many eggs are in the nest?
2. How many are 2 birds and 2 birds?
3. How many ones are there in four?
4. Make 2 short lines; then 2 more. I I I I
5. How many 2's are there in four? 2 and 2 are how many?
6. Count four by ones. Count four by 2's.
7. Two birds taken from four birds leave how many?
8. Four less one are how many? Four less 3?
9. How many are 3 and 1? 1 and 3?
10. How many ones are there in four?
11. How many 2's are there in four?
12. Four are how many more than three? How many more than two?
13. How many eggs must be taken from the nest to leave 1 egg? How many to leave 2? To leave 3?
14. Write the word *four* on your slate. *Four.*
15. Make the figure that stands for *four.* *4.*
16. Count from 4 to one. *Four, three, two, one.*
17. How many are four and one more? ***Five.***

LESSON V.

1. How many birds do you see in the picture? How many peaches? How many birds are on the limb?
2. Five birds are how many more than 4? Than 3?
3. Three birds and 2 birds are how many birds?
4. Two peaches and 3 peaches are how many?
5. Five peaches less 3 peaches are how many?
6. Five peaches less 2 peaches are how many?
7. Two fishes and how many more make five fishes?
8. Three fishes and how many more make five?
9. Make 3 short lines. Now 2 more. III II
10. Three and 2 are how many? 2 and 3?
11. Make 4 short lines. Now one more. IIII I
12. Four and 1 are how many? 1 and 4?
13. If 2 birds fly away, how many are left? How many if 2 more fly away?
14. How many 2's are there in five?

Ans. Two 2's and 1 over.

15. Write the word *five* on your slate. *Five.*
16. Write the figure that stands for *five*. *5.*
17. Count five. Count from five back to one.
18. How many are five and one more? ***Six.***

LESSON VI.

1. There are three boats on the water and three on the land. How many are there in all ? 3 and 3 are how many?

2. How many boys are on the ice? How many 2's are there in six?

3. Three boats from six boats leave how many boats? Two boats from six boats leave how many? Four boats from six boats?

4. Six boys are how many more than 4 boys?

5. How many 2's make six? How many 3's?

6. Count six by ones. By 2's. By 3's.

7. There are 2 sails on 1 boat; how many are there on 3 boats?

8. How many are three 2's? Two 3's?

9. How many are six boats less 5 boats?

10. Four boats and how many more make six boats?

11. Make six lines by 2's. || || || By 3's. ||| |||

12. Write the word *six* on your slate. *Six.*

13. Make the figure that stands for *six.* *6.*

14. Count six. Count from six back to one.

15. How many are six and one more? ***Seven.***

LESSON VII.

1. How many boats are 2 boats and 2 boats?
2. How many boys are 2 boys, 2 boys, and 2 boys?
3. Count by 2's to 4. Count by 2's to 6.
4. How many 2's make 6? How many 3's make 6?
5. Six cents are how many more than 3 cents? How many more than 4 cents? Than 2 cents?
6. Repeat this table.

1 and 5 are 6.	4 and 2 are 6.
2 and 4 are 6.	5 and 1 are 6.
3 and 3 are 6.	6 and 0 are 6.

7.

6 from 6 leaves 0.	3 from 6 leaves 3.
5 from 6 leaves 1.	2 from 6 leaves 4.
4 from 6 leaves 2.	1 from 6 leaves 5.

8. A boy had 6 marbles and lost 3; how many marbles had he left?
9. Six marbles less 4 marbles are how many marbles?
10. Mary had 4 cents and Henry gave her 2 cents more. How many cents had she then?
11. Two cents and 4 cents are how many cents?
12. How many balls put with 2 balls will make 6 balls? How many with 4? How many with 3?
13. How many balls taken from 6 balls will leave 3 balls? How many will leave 2 balls? 5 balls?

How many are

14. 2 boys and 4 boys?
15. 3 books and 3 books?
16. 5 pins and 1 pin?
17. 6 men less 2 men?
18. 5 caps less 3 caps?
19. 6 pinks less 5 pinks?

SEVEN.

LESSON VIII.

1. Four trees and 3 trees are how many trees?

2. If 3 trees are cut down, how many will be left?

3. How many apples are there on the tree? 3 apples and 2 apples and 2 apples are how many?

4. If three apples fall from the tree, how many are left? If 2 more fall, how many are left?

5. Three girls and how many more make seven girls? Four and how many more make seven?

6. Seven bunches of grain are how many more than 5 bunches? Than 3 bunches? Than 2 bunches?

7. How many girls are shown in the picture? If 1 girl leaves, how many will remain? If 3 leave? If 4 leave? If 2 leave? If 6 leave?

8. Make 3 lines on your slate. 3 more. III III

9. How many more will make seven?

10. How many 3's are there in seven and how many over? How many 4's and how many over?

11. Write the word *seven* on your slate. *Seven.*

12. Make the figure that stands for *seven.* *7.*

13. Count seven. Count from seven back to one.

14. How many are seven and one more? ***Eight.***

LESSON IX.

1. How many ones are there in seven? | | | | | | |
2. How many 2's, and how many over?
3. How many 3's, and how many over?
4. Seven girls are how many more than 2 girls? Than 5 girls? Than 1 girl? Than 3 girls? Than 4 girls?
5. Three and how many make 7? 4 and how many? 2 and how many? 5 and how many?
6. Repeat this table.

1 and 6 are 7.	4 and 3 are 7.
2 and 5 are 7.	5 and 2 are 7.
3 and 4 are 7.	6 and 1 are 7.

7.

7 from 7 leaves 0.	3 from 7 leaves 4.
6 from 7 leaves 1.	2 from 7 leaves 5.
5 from 7 leaves 2.	1 from 7 leaves 6.
4 from 7 leaves 3.	0 from 7 leaves 7.

8. James had 7 cents, and gave 5 cents for a pencil. How many cents had he left?
9. George gave 4 peaches to his brother and 3 to his sister. How many did he give to both?
10. How many books put with 2 books will make 7 books? How many books are 3 books and 4 books?
11. How many yards of ribbon cut from 7 yards will leave 5 yards? How many will leave 1 yard? 6 yards? 4 yards? 3 yards? 2 yards?

How many are

12. 4 girls and 3 girls?
13. 2 horses and 5 horses?
14. 3 boxes and 4 boxes?
15. 7 trees less 3 trees?
16. 6 houses less 4 houses?
17. 7 figs less 5 figs?

LESSON X.

1. How many sheep are shown in the picture?

2. There are 4 sheep in one place and 4 in another. How many are there in all? 4 and 4 are how many?

3. Eight arches are how many more than 7? How many more than 6? Than 5? Than 4? Than 3? Than 2?

4. Make eight lines on your slate by 2's. || || || || By 4's. |||| ||||

5. Count eight by 2's. Count eight by 4's.

6. How many 2's are there in eight? How many 4's?

7. Eight sheep less 4 sheep are how many sheep?

8. If 2 arches are taken from eight arches, how many are left? If 5 are taken? If 6 are taken?

9. Four sheep, 3 sheep, and 1 sheep are how many sheep?

10. How many are eight less 6? Eight less 5?

11. Write the word *eight* on your slate. *Eight.*

12. Make the figure that stands for *eight.* *8.*

13. Count eight. Count from eight back to one.

14. How many are eight and one more? ***Nine.***

LESSON XI.

1. How many feet has a sheep? How many feet have 2 sheep? How many 4's are there in 8?

2. How many hands has one boy? How many have 2 boys? 3 boys? 4 boys?

3. How many boys must hold up both hands, to show 8 hands? How many 2's are there in 8?

4. Eight are how many more than 7? Than 5? Than 2? Than 6? Than 4?

5. Eight less 6 are how many? 7 less 3?

6. Repeat this table.

0 and 8 are 8.	4 and 4 are 8.
1 and 7 are 8.	5 and 3 are 8.
2 and 6 are 8.	6 and 2 are 8.
3 and 5 are 8.	7 and 1 are 8.

7.

8 from 8 leaves 0.	4 from 8 leaves 4.
7 from 8 leaves 1.	3 from 8 leaves 5.
6 from 8 leaves 2.	2 from 8 leaves 6.
5 from 8 leaves 3.	1 from 8 leaves 7.

8. There are 2 red cars, 1 blue car, and 5 yellow cars in a train. How many are there in all?

9. If there are 8 horses in a stable and 3 are taken out, how many will be left? If 5 are taken out? If 1 is taken out?

How many are

10. 4 cows and 4 cows?
11. 6 hens and 2 hens?
12. 1 pail and 7 pails?
13. 3 words and 5 words?
14. 8 men less 1 man?
15. 8 figs less 5 figs?
16. 8 rings less 7 rings?
17. 8 dogs less 3 dogs?

LESSON XII.

1. How many roses are shown in the picture? How many acorns?

2. How many acorns are on the upper branch? How many are on the lower branch? How many are on both branches?

3. Four and 5 are how many? 5 and 4 are how many? 6 and 3 are how many?

4. Nine less 5 are how many? Nine less 4?

5. How many 3's are there in 6? How many in 9?

6. Nine are how many more than 6? 6 than 3?

7. If 5 acorns drop from the branch, how many are left? If 4 more drop how many are left?

8. If 3 roses are picked, how many are left? If 3 more are picked? If still 3 more are picked?

9. Make nine lines on your slate by 3's. ||| ||| |||

10. Count nine by ones. Count nine by 3's.

11. How many 4's in nine, and how many over?

12. Write the word *nine* on your slate. *Nine.*

13. Make the figure for *nine*. *9.*

14. Count nine. Count back from nine to one.

15. How many are nine and one more. ***Ten.***

LESSON XIII.

1. Make nine short lines on your slate. |||||||||

2. How many ones are there in 9? How many 3's?

3. If 3 girls have 3 roses each, how many have they all?

4. How many must be taken from 9 to leave 6? To leave 4? To leave 5? To leave 1? To leave 7? To leave 2? To leave 3? To leave 8?

5. Repeat this table.

0 and 9 are 9.	5 and 4 are 9.
1 and 8 are 9.	6 and 3 are 9.
2 and 7 are 9.	7 and 2 are 9.
3 and 6 are 9.	8 and 1 are 9.
4 and 5 are 9.	9 and 0 are 9.

6.

9 from 9 leaves 0.	4 from 9 leaves 5.
8 from 9 leaves 1.	3 from 9 leaves 6.
7 from 9 leaves 2.	2 from 9 leaves 7.
6 from 9 leaves 3.	1 from 9 leaves 8.
5 from 9 leaves 4.	0 from 9 leaves 9.

7. There are 9 leaves on two branches. If 4 leaves are on one of the branches, how many are there on the other branch?

8. A boy had 5 peaches in one pocket, and 4 in another. How many had he in both pockets?

9. If he gives away 2 peaches out of each pocket, how many will he have left?

How many are

10. 7 tops and 2 tops?
11. 3 dollars and 6 dollars?
12. 9 eggs less 2 eggs?
13. 7 birds less 5 birds?

LESSON XIV.

1. Make 5 short lines; then 5 more lines. ||||| |||||

2. How many ones make ten? How many 5's?

3. In the picture, how many books are on the upper shelf of the bookcase? How many are on the lower shelf?

4. How many cherries are there on the branch?

5. If 2 cherries are picked, how many will remain? If 2 more are picked, how many will be left? If still 2 more are picked?

6. Ten cherries less 6 cherries are how many cherries? Ten cherries less 8 are how many?

7. Make ten short lines by 2's. || || || || ||

8. Count ten by 2's. Count ten by 5's.

9. How many flower-pots are on the ground? How many are on the stand? How many are there in all? Six pots and 4 pots are how many pots?

10. Name each of the numbers that can be expressed by a single figure.

11. How is the number *ten* expressed?

12. Write the word *ten* on your slate. *Ten.*

13. Make the figures to express *ten.* *10.*

LESSON XV.

1. Make ten short lines on your slate. ||||||||||

2. Ten are how many ones? How many 2's? 5's?

3. How many 4's are there in 10, and how many over? How many 3's, and how many over?

4. How many must be taken from 10 to leave 5? To leave 4? To leave 7? To leave 3? To leave 2?

5. John had 6 marbles and bought 4 more. How many had he then?

6. Mary had 10 cents and gave 3 cents for a pencil. How many had she left?

7. Repeat this table.

0 and 10 are 10.	5 and 5 are 10.
1 and 9 are 10.	6 and 4 are 10.
2 and 8 are 10.	7 and 3 are 10.
3 and 7 are 10.	8 and 2 are 10.
4 and 6 are 10.	9 and 1 are 10.

8.

10 from 10 leaves 0.	5 from 10 leaves 5.
9 from 10 leaves 1.	4 from 10 leaves 6.
8 from 10 leaves 2.	3 from 10 leaves 7.
7 from 10 leaves 3.	2 from 10 leaves 8.
6 from 10 leaves 4.	1 from 10 leaves 9.

How many are

9. 5 pins and 5 pins?
10. 3 trees and 7 trees?
11. 6 birds and 4 birds?
12. 2 sheep and 8 sheep?
13. 9 plums less 5 plums?
14. 10 pears less 7 pears?
15. 8 pens less 5 pens?
16. 10 figs less 9 figs?

In all these lessons of counting in *series* the teacher should use objects, or the numeral frame, until the pupil thoroughly understands the process.

LESSON XVI.

1. How many are ten balls and one ball more? ***Eleven.*** Written *11*.

2. How many are eleven balls and one ball? ***Twelve.*** Written *12*.

3. Twelve balls and one more are ***Thirteen.*** Written *13*.

4. Thirteen balls and one more are ***Fourteen.*** Written *14*.

			Written
5.	Fourteen and one more are	***Fifteen.***	*15*
6.	Fifteen and one more are	***Sixteen.***	*16*
7.	Sixteen and one more are	***Seventeen.***	*17*
8.	Seventeen and one more are	***Eighteen.***	*18*
9.	Eighteen and one more are	***Nineteen.***	*19*
10.	Nineteen and one more are	***Twenty.***	*20*

11. How many *units* are there in the number one? In the number ten? Write the figures for ten. *10*

12. What does the 0, or *cipher*, denote?

Ans. No *units*.

13. What does the 1 denote? *Ans.* One *ten*.

14. Then, what do the figures 10 denote?

Ans. 1 *ten* and 0 *units*, or *ten*.

LESSON XVII.

When two figures are written side by side, what does each figure denote? The one on the *right* denotes **units,** and the one on the *left* denotes **tens.**

1. What do the figures 11 denote?

Ans. 1 *ten* and 1 *unit*, or *eleven*.

2. What do the figures 12 denote?

3. How many *tens* are there in twenty?

4. What do the figures 20 denote?

Ans. 2 *tens* and 0 *units*, or *twenty.*

5. What then do the figures 21 denote? Written

6. Two tens and two units are ***Twenty-two.*** *22*

7. Two tens and three units are ***Twenty-three.*** *23*

8. Two tens and four units are ***Twenty-four.*** *24*

9. Two tens and five units are ***Twenty-five.*** *25*

10. Two tens and six units are ***Twenty-six.*** *26*

11. Two tens and seven units are ***Twenty-seven.*** *27*

12. Two tens and eight units are ***Twenty-eight.*** *28*

13. Two tens and nine units are ***Twenty-nine.*** *29*

14. *Three* tens and no units are ***Thirty.*** *30*

15. Count from 1 to 30. Write the numbers from 1 to 30.

LESSON XVIII.

1. How many *units* are ten? Twenty? Thirty?

2. How many *tens* are ten? Twenty? Thirty?

3. Write the figures that stand for thirty-one. For thirty-two. For thirty-three.

4. What do the figures 34 denote?

5. Count 30. Count from 30 to 40. Write the numbers from 30 to 40.

6. How many *units* are four tens? How many *tens* are forty? Make the figures that stand for forty. *40*

7. Write the figures for forty-three. For forty-four. For forty-five.

8. Write the numbers from forty to fifty.

9. How many tens are fifty? How many units?

10. Write the figures that stand for fifty. *50*

11. What do the figures 56 denote?
12. Count from fifty to sixty.
13. Write the numbers from fifty to sixty.
14. How many *tens* are sixty? How many *units?*
15. Write the figures that stand for sixty. *60*
16. What do the figures 67 denote?

LESSON XIX.

1. Count from sixty to seventy. Write the numbers from sixty to seventy.
2. How many *tens* are seventy? How many *units?*
3. Write the figures that stand for seventy. *70*
4. Write the figures for seventy-five.
5. What do the figures 77 denote? 78?
6. Make the figures denoting seventy-two. Sixty-five. Seventy-four. Sixty-three. Seventy-one.
7. Write the numbers from seventy to eighty.
8. How many *tens* are eighty? How many *units?*
9. Write the figures that stand for eighty. *80*
10. For eighty-one. For eighty-two.
11. What do the figures 87 denote?
12. Write the numbers from eighty to ninety.
13. How many *tens* are ninety? How many *units?*
14. Write the figures that stand for ninety. *90*
15. Write the figures from 90 to 100.
16. What do the figures 99 denote?

The greatest number that can be expressed by two figures is 99. Ninety-nine and one more are **one hundred.** Written *100.*

17. What does 1 with two 0's on the right mean?

LESSON XX.

PLUS AND EQUAL SIGNS.

Make two lines to *cross* each other; thus, +.

Because + shows what is to be done, it is called a **Sign.** It is named **Plus,** and *plus* means *more.*

The sign + is used in place of the word *and.*

Thus, instead of writing 2 *and* 3 are 5, we may write 2 + 3 are 5, which means 2 *and* 3 *more* are 5; and is read 2 *plus* 3 are 5.

Make two horizontal lines; thus, =.

Since these two lines = are equal in length, we will hereafter use them to mean *equal,* in place of the word *equal,* or of the word *are.*

Thus, instead of writing 3 *and* 2 *are* 5, or 3 + 2 *equal* 5, we may write 3 + 2 = 5.

Because these two lines mean *equal,* they are called the **Sign of Equality.**

1. Copy and read the following:

$5+4=9$ $2+5=7$ $4+4=8$ $6+4=10$

2. Write the following, using figures and signs:

Three and five equal eight. Six and three equal nine.

LESSON XXI.

ADDITION.

Counting two or more numbers of the same kind together, so as to find what number they all equal, is called **adding,** or **Addition.**

The *Plus Sign,* +, is the **Sign of Addition.**

The sign + shows that what is written at the right of it is to be *added* to what is written before it.

Thus 4 + 2 means that 2 is to be *added* to 4.

The number obtained by adding is called the **Sum.**

Thus, 7 is the *sum* of 3 and 4, or 3 + 4.

Usually, when numbers are to be *added*, they are written in a *vertical* line or column.

1. *Add* the numbers 2, 3, and 4, writing them as shown in the margin, and draw a line under the column.

We first find that 4 and 3 are seven, and next that 7 and 2 more are 9, which we write below the line for the *sum* of 2, 3, and 4.

	2
	3
	4
Sum	9

2. In like manner copy and add the following:

1	3	2	1	2	3	2	4	3	2
2	4	3	3	5	3	5	3	4	3
3	2	1	4	1	4	2	1	3	2

LESSON XXII.

1. If a spool of thread costs 6 cents and a yard of tape 4 cents, how much do both cost?

2. Henry rode 7 miles and walked 2 miles. How far did he go?

3. A tailor sold 5 yards of cloth at one time and 5 yards at another. How many yards did he sell in all?

4. Carrie had 4 roses and Nellie had 3. How many roses had both?

5. In a fruit-dish there are 5 red apples and 4 green ones. How many apples are there in the dish?

6. Two birds sit on one limb, 3 on another, and 5 on another. How many birds are there in all?

7. A man paid 3 dollars for a cord of wood, and 6 dollars for a ton of coal. How much did he pay for both?

8. Mary bought some ribbon for 7 cents, and some buttons for 3 cents. How much did she pay for both?

9. Begin with 1 and count 9 by 2's; thus, *one, three, five, seven, nine.*

10. Begin with 1 and count 10 by 3's; thus, *one, four, seven, ten.*

11. Copy and add the following on your slate:

2	2	2	3	2	1	1	1	3
2	2	1	3	3	2	3	1	1
2	2	2	2	1	1	2	2	1
2	2	1	1	2	3	1	2	2
2	1	2	1	1	1	2	3	1

LESSON XXIII.

MINUS SIGN.

Make a short horizontal line on your slate; thus, —.

A line written thus —, between two numbers, is used to mean *less*, in place of the word *less*.

Thus, instead of writing 2 from 5 leaves 3, or 5 *less* 2 *equals* 3, we may write $5 - 2 = 3$.

The sign — is named **Minus**, and *minus* means *less*.

1. Read the following:

$8 - 2 = 6$ $\quad 9 - 5 = 4$ $\quad 10 - 4 = 6$ $\quad 8 - 6 = 2$

$7 - 3 = 4$ $\quad 6 - 6 = 0$ $\quad 7 - 4 = 3$ $\quad 10 - 3 = 7$

2. Write the following, using figures and signs:

Nine less two equals seven. Six less four equals two.

Seven less two equals five. Ten less five equals five.

3. Copy the following, writing the *result* in place of the question mark (?).

$8 - 1 = ?$ $\quad 5 - 4 = ?$ $\quad 5 - 5 = ?$ $\quad 10 - 2 = ?$

$7 - 5 = ?$ $\quad 10 - 6 = ?$ $\quad 9 - 6 = ?$ $\quad 9 - 8 = ?$

LESSON XXIV.

SUBTRACTION.

Taking one number from another of the same kind is called **subtracting,** or **Subtraction.**

The *Minus Sign*, −, is the **Sign of Subtraction.**

The sign − shows that what is written at the right is to be *taken from* what is written before it.

Thus, 6 − 4 means that 4 is to be *subtracted* from 6.

The number that shows how many *remain* after subtracting is called the **Remainder,** sometimes the **Difference.**

Thus, 5 is the *remainder* after taking 4 from 9, or 5 is the *difference* between 9 and 4, written 9 − 4 = 5.

1. Find the *remainder* of

7 less 6	10 less 6	8 − 4	7 − 7
9 less 3	6 less 4	9 − 7	10 − 7

In performing subtraction, the numbers when written are usually arranged as in the margin, the smaller under the greater, with a line drawn underneath. We say four from 7 leaves 3, which we write below the line for the *remainder.*

$$\begin{array}{r} 7 \\ 4 \\ \hline 3 \end{array}$$

2. Copy and write the remainder in each of the following:

5	6	8	9	6	4	8	10	8	9
3	3	5	9	5	2	3	8	7	4
—	—	—	—	—	—	—	—	—	—

LESSON XXV.

✱✱✱ ✱✱✱ ✱✱✱ ✱✱✱

1. Here is a picture of twelve stars.
2. Count twelve. Make the figures for twelve. *12*
3. Count from 12 back to 1.
4. Count by 2's to 8. Count from 8 back to 0.

5. How many are 8 less 2? 6 less 2? 4 less 2? 2 less 2?

6. Count by 2's to 10. Count from 10 back to 0.

7. How many are 10 less 2? 9 less 2? 7 less 2?

8. Count by 2's to 12. Count from 12 back to 0.

9. Count by 3's to 9. Count from 9 back to 0.

10. How many are 7 less 3? 8 less 3? 3 less 3?

11. Count by 3's to 12. Count from 12 back to 0.

12. How many are 12 less 3? 9 less 3? 6 less 3?

13. How many 3's are there in 12? How many 2's?

14. If there are 12 sheep in a yard and 3 jump out, how many sheep are left? $12 - 3 = ?$

15. If 3 more jump out, how many are left? $9 - 3 = ?$

16. If 3 more jump out, how many are left? $6 - 3 = ?$

17. If 3 more jump out, how many are left? $3 - 3 = ?$

18. Write the following, using the proper *signs* and the correct number in the place of the (?).

3 and 3 and 3 are 9.	9 less 5 equal 4.
2 and 3 and 3 = ?	10 less 7 = ?
3 and 1 and 3 are 7.	5 from 8 leaves ?

LESSON XXVI.

1. Count by 2's from 1 to 11. From 1 to 13.

2. Count by 3's from 1 to 10. From 1 to 13.

3. Count by 3's from 1 to 11. From 2 to 14.

4. Count by 4's from 1 to 9. From 2 to 10.

5. Count by 4's from 2 to 14. From 3 to 15.

6. Count and find the sum of:

3	3	3	2	4	4
3	4	3	2	4	3
3	3	3	2	4	2
3	1	2	2	4	1

Find the remainder of:

7	9	6	8	7
3	4	6	3	6
9	10	11	5	12
3	4	7	2	8

7. Find and write the *result* in each of the following:

$4+4+1=?$	$2+2+2+2+2=?$	$10-7=?$
$3+3+3+3=?$	$3+3+3+2+1=?$	$5-5=?$
$2+4+3+1=?$	$3+1+4+2+1=?$	$9-6=?$

LESSON XXVII.

1. Four and four are how many? 8 and 4? 12 and 4?
2. Count 12 by 4's. How many 4's are there in 12?
3. Four from 12 leaves how many? 4 from 8?
4. Twelve are how many more than 8? More than 4?
5. Four and how many make 8? 8 are how many less than 12?
6. Count by 4's from 1 to 17. From 2 to 18. From 3 to 19. From 0 to 20.
7. How many are 20 less 4? 16 less 4? 12 less 4?

When the *Sum* of two or more numbers, or the *Difference* of two numbers, is named instantly, it is called addition or subtraction, *at sight.*

Thus, in the example 4 + 3, say 7, instead of 4 and 3 are 7.

8. Add at sight:

4	4	8	5	3
4	3	4	3	4

9. Subtract at sight:

8	7	8	9	6
4	3	5	5	3

LESSON XXVIII.

1. Four are how many less than 9? 9 are how many more than 5?
2. How many are 12 less 7? 12 less 5 are how many?
3. Ira gave 5 cents for a pencil and 4 cents for a top. How much did both cost?
4. John has 6 books and Jane has 5 books. How many books have both?

5. James gave 7 cents for a writing-book and had 5 cents left. How many cents had he at first?

6.

$8+4=?$	$3+5+2=?$	$10-5=?$
$7+5=?$	$4+5+2=?$	$11-3=?$
$8+3=?$	$6+4+4=?$	$12-4=?$

7. Asa is 9 years old. How old will he be 4 years hence?

8. Martha is 12 years old; her sister is 4 years younger. How old is her sister?

9. There are 11 geese and 4 ducks swimming on the pond. How many are there of both? How many more geese than ducks are there?

10. One boy had 7 cents; another had 4. How many cents did both have?

11. Copy and add:

4	4	4	4	4	4	3	5	2	1
4	4	4	4	3	4	4	2	4	2
4	4	4	4	5	3	2	4	3	4
4	3	2	1	3	2	1	2	4	5
—	—	—	—	—	—	—	—	—	—

LESSON XXIX.

1. How many are 5 cents and 5 cents?
2. How many are 10 cents and 5 cents more?
3. How many are 15 cents and 5 cents more?
4. Count 10 by 5's. Count 15 by 5's. Count 20 by 5's.
5. How many 5's are there in 10? In 15? In 20?
6. How many are

$5+5?$	$2+5+5?$	$20-5?$	$14-5?$
$4+5?$	$3+5+5?$	$15-5?$	$13-5?$
$3+5?$	$4+5+5?$	$10-5?$	$12-5?$

7. Count by 5's from 15 back to 0. From 20 back to 0.
8. Count by 5's to 25. From 25 back to 0.

9. Copy, count, and write the *sum* of the following:

5	5	5	5	5	1	2	3	2	4
5	5	5	5	5	5	5	2	5	1
5	1	2	3	4	5	5	5	4	5

LESSON XXX.

1. How many 5's are there in 10? How many 2's?
2. How many 5's are there in 15? How many 3's?
3. How many 5's are there in 20? How many 4's?
4. Count 25 by 2's, beginning with 1. Count 25 by 5's.
5. Begin with 4 and count by 3's to 25.
6. How many are 8 boys and 4 boys? 12 boys less 4 boys? 13 boys less 5 boys?
7. How many are 6 girls and 5 girls? 11 girls less 5 girls? 12 girls and 5 girls?
8. How many are 15 marbles less 5 marbles? 14 less 5? $15-5$? $14-5$?

9. Add at sight:

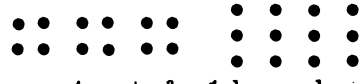
5	3	2	9	5
4	7	9	5	10

10. Subtract at sight:

9	10	10	14	15
4	7	6	4	5

LESSON XXXI.

MULTIPLICATION.



1. If you pay 4 cents for 1 lemon, how many times 4 cents must you pay for 3 lemons? $4+4+4=?$ How many are three 4's?

Instead of writing $4+4+4=12$, we may write 3 *times* 4 are 12.

2. If 1 orange costs 3 cents, how many times 3 cents must you give for 4 oranges? 3+3+3+3=? How many are four 3's?

Instead of writing 3+3+3+3=12, we may write 4 *times* 3 are 12.

Make two lines to cross each other; thus, ×.

This sign, ×, is used in place of the word *times*.

Thus, instead of writing 3 *times* 4 are 12, or 4 *times* 3 are 12, we may write 4 × 3 are 12, or 3 × 4 = 12.

3. Read the following:

2 × 3 are 6	4 × 3 are 12	4 × 2 = 8
3 × 3 are 9	3 × 4 are 12	5 × 2 = 10

LESSON XXXII.

1. If you take 4 peaches 3 times from a fruit-dish, how many peaches will you take in all?

2. How many peaches are 3 times 4 peaches?

Taking one of two numbers as many times as there are ones or units in another is called **Multiplication.**

The × is called the **Sign of Multiplication.** It is read *times*, or *multiplied by*.

Thus, 3 × 2 is read 3 *multiplied by* 2, or 2 *times* 3; 4 × 3 is read 4 *multiplied by* 3, or 3 *times* 4.

The number obtained by multiplying one number by another is called the **Product.**

MULTIPLICATION TABLE.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25

LESSON XXXIII.

1. How many oranges are 5 times 3 oranges?

There are two ways of solving this example.

FIRST. Addition.	SECOND. Multiplication.
3	3
3	5
3	15 Product.
3	
3	
15 Sum.	

FIRST. — We may write the figure 3 *five* times in a column, and draw a short line under it. Then we count, or add, thus: 3, 6, 9, 12, 15, and write the result below the line.

SECOND. — We may write the figure 3 but *once* and write the figure 5 under it, to show *how many times* 3 is to be taken, and draw a short line. Then we say 5 times 3 are 15, and write the result below the line. The result is 15 in both cases, but in the first case it is obtained by *addition* and is called the *Sum;* in the second case, it is obtained by *multiplication* and is called the *Product.*

Hence, *multiplication* is also a short method of *adding equal numbers.*

Find the result by addition and by multiplication:

2. Of 2 times 5 cents.
3. Of 4 times 3 figs.
4. Of 5 times 4 hats.
5. Of 5 times 2 boys.

LESSON XXXIV.

1. How many peaches are there on 3 plates, if there are 5 peaches on each plate?

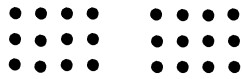
SOLUTION. — Since there are 5 peaches on one plate, on 3 plates there are 3 times 5 peaches, which are 15 peaches.

2. If one peach costs 4 cents, how much will 3 cost?
3. If 1 orange costs 5 cents, what will 2 oranges cost?
4. How much will 4 pencils cost, if 1 costs 5 cents?
5. How many eggs are there in 3 nests, if there are 3 eggs in each nest?
6. At 5 cents each, how much will 5 tops cost?
7. Multiply at sight:

5×2; 4×3; 3×5; 4×4; 4×5; 5×4; 5×5.

LESSON XXXV.

DIVISION.



1. I had 12 marbles and gave 4 to Rob. How many had I left? I gave 4 more to Tom. How many had I left? I gave 4 more to Asa. How many had I left? How many times 4 marbles did I give away?

2. How many are 12 less 4? 8 less 4? 4 less 4?

3. How many times can 4 marbles be taken from 12 marbles? How many 4's are there in 12?

Instead of saying that 4 can be taken from 12, 3 *times*, or that there are 3 *fours* in 12, say, 4 is *contained in* 12, 3 times.

Make two dots, one above and one below a horizontal line; thus, ÷. This sign is used in place of the words *divided by*.

Thus, instead of writing 12 *divided by* 4, write 12 ÷ 4.

Express by figures and the proper signs:

4. Two is contained in twelve, six times.
5. Four is contained in twenty, five times.
6. Five is contained in twenty-five, five times.

LESSON XXXVI.

1. How many cups, at five cents each, can you buy for 15 cents? How many times can 5 cents be taken from 15 cents? 5 is contained in 15 how many times? 15÷5=?

Finding how many times one number is contained in another is called **Division.**

The sign, $\div$, is called the **Sign of Division.**

Thus, $15 \div 5$ is read, 15 *divided by* 5.

The number obtained by dividing one number by another is called the **Quotient.**

DIVISION TABLE.

$2 \div 2 = 1$	$3 \div 3 = 1$	$4 \div 4 = 1$	$5 \div 5 = 1$
$4 \div 2 = 2$	$6 \div 3 = 2$	$8 \div 4 = 2$	$10 \div 5 = 2$
$6 \div 2 = 3$	$9 \div 3 = 3$	$12 \div 4 = 3$	$15 \div 5 = 3$
$8 \div 2 = 4$	$12 \div 3 = 4$	$16 \div 4 = 4$	$20 \div 5 = 4$
$10 \div 2 = 5$	$15 \div 3 = 5$	$20 \div 4 = 5$	$25 \div 5 = 5$

LESSON XXXVII.

1. Among how many boys can 15 cents be divided if each boy is to receive 5 cents?

There are two ways of solving this example:

FIRST.
By Subtraction.

15 cents.
5
10 cents.
5
5 cents.
5
0 cents.

FIRST. — Taking 5 cents from 15 cents, 10 cents are left; again, taking 5 cents from 10 cents, 5 cents are left; again, taking 5 cents from 5 cents, no cents are left. Since 5 cents have been subtracted or taken from 15 cents 3 *times*, I gave the 15 cents to 3 boys.

SECOND. — Since we wish to find how many times 5 cents are contained in 15 cents, we write the expression thus, $15 \div 5 = 3$, and read it, 15 *divided by* 5 *equals* 3. Or, we write 15, and on the left write 5, with a short curved line between them, and a short line under the 15. Then we say, 5 *is contained* in 15, 3 *times*, and write the result below the line.

SECOND.
By Division.

$5)\underline{15}$
3

The result is 3 in both cases, but in the first case it is obtained by *subtraction*, in the second case by *division*. Hence, *division* is also a short method of performing several *subtractions* of the *same number*.

Find the result, both by subtraction and by division, of each of the following:

How many

2. 3's in 12?	4. 4's in 20?	6. 5's in 20?
3. 4's in 16?	5. 3's in 15?	7. 5's in 25?

LESSON XXXVIII.

1. How many times 2 cents are there in 10 cents? In 12 cents? In 8 cents?

2. How many times 3 days are there in 12 days?

3. How many times 4 plums are there in 8 plums? In 12 plums? In 16 plums?

4. How many times 5 hours are there in 15 hours? In 20 hours? In 25 hours?

5. At 5 cents each, how many toys can be bought for 20 cents?

SOLUTION. — As many toys as 5 cents are contained times in 20 cents, which are 4 times. Hence 4 toys can be bought for 20 cents.

6. How many quarts of milk, at 4 cents a quart, can be bought for 16 cents?

7. To how many boys can you give 12 apples, if you give them 4 apples apiece?

8. How many hats can be bought for 25 dollars, at 5 dollars apiece?

9. Ella paid 20 cents for some ribbon, at 5 cents a yard. How many yards did she buy?

10. Clarence gave 16 cents for some tops, at 4 cents apiece. How many tops did he buy?

11. Divide at sight:

$8 + 2 = ?$	$8 \div 4 = ?$	$15 \div 5 = ?$	$15 \div 3 = ?$	
$6 + 3 = ?$	$10 \div 5 = ?$	$10 \div 2 = ?$	$16 \div 4 = ?$	
$4)\underline{20}$	$5)\underline{25}$	$4)\underline{12}$	$5)\underline{20}$	$3)\underline{9}$	$2)\underline{12}$

LESSON XXXIX.

EQUAL PARTS OF NUMBERS.

1. If 6 oranges are divided equally between 2 girls, into how many *equal parts* are the oranges divided?

When a number is divided into 2 equal parts, one of the parts is called **one half** of the number.

2. How many are *one half* of 6 stars?

$6 \div 2 = 3$, or *one half* of 6 is 3.

3. How many are one half of 8 boys? Of 10 boys?

4. If you put 10 bushels of apples into 2 boxes, what part of the whole do you put into 1 box? How many bushels?

Instead of finding how many times one number is *contained in* another of the *same* kind, it is sometimes required to divide a number into **equal parts.** The operation in both cases is the same, but the reasoning is somewhat different.

5. At 3 cents each, how many pears can be bought for 6 cents?

SOLUTION. — As many pears as 3 cents are contained times in 6 cents, which are 2 *times.* Hence 2 pears can be bought for 6 cents. $6 \div 3 = 2$.

6. If 2 pears cost 6 cents, what will 1 pear cost?

SOLUTION. — If 2 pears cost 6 cents, 1 pear will cost *one half* of 6 cents, which is 3 cents. $6 \div 2 = 3$.

7. If 12 cherries are divided equally among 3 boys, into how many *equal parts* are they divided?

When a number is divided into 3 equal parts, one of the parts is called **one third** of the number.

8. How many cherries are *one third* of 12 cherries?

$12 \div 3 = 4$, or *one third* of 12 is 4.

9. How many cents are one third of 9 cents?

10. How many marbles are one third of 15 marbles?

11. If 12 pinks grow upon 4 stems, each containing an equal number, how many grow upon each stem?

When a number is divided into four equal parts, one of the parts is called **one fourth** of the number.

12. How many pinks are *one fourth* of 12 pinks?

$12 \div 4 = 3$, or *one fourth* of 12 is 3.

13. How many dollars are one fourth of 8 dollars?

14. What is one fourth of 12? Of 16? Of 20?

When a number is divided into 5 equal parts, one of the parts is called **one fifth** of the number.

15. How many balls are *one fifth* of 20 balls?

$20 \div 5 = 4$, or *one fifth* of 20 is 4.

16. How many cents are one fifth of 15 cents?

LESSON XL.

One half is written thus, $\frac{1}{2}$. $\frac{1}{2}$ of 6 is $6 \div 2 = 3$.
One third is written $\frac{1}{3}$. $\frac{1}{3}$ of 6 is $6 \div 3 = 2$.
One fourth is written $\frac{1}{4}$. $\frac{1}{4}$ of 8 is $8 \div 4 = 2$.
One fifth is written $\frac{1}{5}$. $\frac{1}{5}$ of 15 is $15 \div 5 = 3$.

1. How many *halves* are there in any thing? How many *thirds*? How many *fourths*? How many *fifths*?

2. What is $\frac{1}{2}$ of 4 books? $\frac{1}{2}$ of 10 miles? $\frac{1}{2}$ of 8 houses?

3. What is $\frac{1}{3}$ of 6 sheep? $\frac{1}{3}$ of 9 weeks? $\frac{1}{3}$ of 12 dollars?

4. What is $\frac{1}{4}$ of 4 men? $\frac{1}{4}$ of 16 pounds? $\frac{1}{4}$ of 20 chairs?

5. What is $\frac{1}{5}$ of 10 nuts? $\frac{1}{5}$ of 15 trees? $\frac{1}{5}$ of 25 cups?

6. How do you obtain one half of a number? One third? One fourth? One fifth?

7. If 20 marbles are divided equally among 4 boys, how many marbles will each boy receive?

SOLUTION. — Since 20 marbles are divided equally among 4 boys, one boy will receive *one fourth* of 20 marbles, or 5 marbles.

8. If 3 books cost 15 cents, what will 1 book cost?

9. Write on your slates, in a column, and read, all the numbers from 10 to 20. From 20 to 30. From 30 to 40.

10. In a similar manner write and read the numbers from 40 to 50. From 50 to 60. From 60 to 70. From 70 to 80. From 80 to 90. From 90 to 100.

11. How many figures are required in writing each number from 9 to 99?

LESSON XLI.

1. How many are 5 and 1? 3 and 3? 4 and 2?

2. How many are $2+6$? $4+6$? $5+6$? $3+6$? $6+6$? $12+6$? $7+6$? $6+8$? $9+6$? $10+6$?

3. How many are $18+6$? $24+6$? $6+11$?

4. How many are three 6's? Four 6's? Five 6's?

5. Count by 6's to 12. To 18. To 24. To 30.

6. Begin with 1 and count by 6's to 13. To 19. To 25. To 30. To 31.

7. Begin with 2 and count by 6's to 20. To 26. To 32. To 36.

8. How many are $6+6+6$? $4+6+6$? $2+6+6$? $5+6+6$? $3+6+6$? $1+6+6$?

9. How many are $7-6$? $10-6$? $14-6$? $15-6$? $30-6$? $18-6$? $9-6$? $8-6$? $12-6$? $11-6$? $6-6$? $13-6$? $24-6$? $16-6$? $17-6$?

10. How many are 3×6? 4×6? 2×6? 6×5? 6×4? 5×6? 6×6? 6×3? 6×2?

11. How many are $18\div 6$? $30\div 6$? $12\div 6$? $6\div 6$? $24\div 6$? $36\div 6$?

12. Copy and add or count the following:

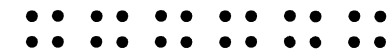
6	6	6	6	6	6	6	5	4	2
6	6	6	6	6	6	5	6	6	4
6	5	4	3	2	1	4	3	2	5
—	—	—	—	—	—	—	—	—	—

LESSON XLII.

1. If 18 figs are equally divided among 6 boys, into how many *equal parts* are the figs divided?

When a number is divided into 6 equal parts, one of the parts is called **one sixth** of the number.

2. How many dots are *one sixth* of 24 dots?



$24\div 6=4$, or *one sixth* of 24.

3. If 24 plants are set in 6 equal rows, what part of 24 plants is set in 1 row? How many plants?

One sixth is written thus, $\frac{1}{6}$. $\frac{1}{6}$ of 24 is $24\div 6=4$.

$\frac{1}{6}$ of $6=1$ $\quad$ $\frac{1}{6}$ of $18=3$ $\quad$ $\frac{1}{6}$ of $30=5$

$\frac{1}{6}$ of $12=2$ $\quad$ $\frac{1}{6}$ of $24=4$ $\quad$ $\frac{1}{6}$ of $36=6$

4. How many are $\frac{1}{6}$ of 12 boys? Of 24? Of 30?

5. If a man has 18 dollars, and he earns 6 more, how many dollars will he then have? $18+6=?$

6. A man having 18 dollars gave 6 dollars for a barrel of flour. How many dollars had he left? $18-6=?$

7. A laborer received 3 dollars a day for 6 days' work. How many dollars did he receive in all? $3 \times 6 = ?$

8. At 3 dollars apiece, how many chairs can be bought for 18 dollars? $18 \div 6 = ?$

9. How many are $\frac{1}{6}$ of 18 pictures? $\frac{1}{6}$ of $18 = ?$

10. Add at sight:

3	4	4	4	6	6
4	5	6	2	4	4
6	3	5	5	3	4

11. Subtract at sight:

10	12	11	13	9	14
6	6	6	6	6	6

12. Multiply at sight:

6	3	6	4
4	6	5	6

13. Divide at sight:

6)24 6)18 6)36 6)42

LESSON XLIII.

1. How many are $3+4$? $4+3$? $5+2$? $2+7$? $3+7$? $4+7$? $5+7$? $6+7$? $1+7$? $7+7$?

2. How many are $7+9$? $7+6$? $7+4$? $7+8$?

3. How many are 14 and 7 more? $14+7=?$

4. Count 21 by 3's. Count 21 by 7's.

5. How many are 21 and 7 more? $21+7=?$

6. Count 28 by 2's. Count 28 by 4's. Count 28 by 7's.

7. Count 35 by 5's. Count 35 by 7's.

8. How many are $12-7$? $15-7$? $14-7$? $8-7$? $10-7$? $11-7$? $7-7$? $9-7$? $13-7$? $35-7$? $28-7$?

9. How many are 35 and 7 more? 42 and 7 more?

10. Count 42 by 2's. By 3's. By 6's. By 7's.

11. How many are 7×2? 3×7? 7×4? 5×7? 7×6? 7×7? 8×7?

12. How many are $21 \div 7$? $28 \div 7$? $49 \div 7$? $56 \div 7$? $42 \div 7$? $35 \div 7$? $14 \div 7$? $7 \div 7$?

LESSON XLIV.

1. If 21 yards of cloth are made into 7 coats of the same size, into how many *equal parts* must they be cut?

When a number is divided into 7 equal parts, one of the parts is called **one seventh** of the number.

2. How many yards are *one seventh* of 21 yards?

$21 \div 7 = 3$, or *one seventh* of 21 is 3.

3. How many birds are one seventh of 14 birds?

One seventh is written thus, $\frac{1}{7}$. $\frac{1}{7}$ of 28 is $28 \div 7 = 4$.

$\frac{1}{7}$ of $7 = 1$ $\frac{1}{7}$ of $21 = 3$ $\frac{1}{7}$ of $35 = 5$ $\frac{1}{7}$ of $49 = 7$

$\frac{1}{7}$ of $14 = 2$ $\frac{1}{7}$ of $28 = 4$ $\frac{1}{7}$ of $42 = 6$ $\frac{1}{7}$ of $56 = 8$

4. How many are $\frac{1}{7}$ of 14 plates? Of 28? Of 35? Of 42? Of 21? Of 49?

5. John gave 7 cents for a pencil, 2 cents for a top, and had 2 cents left. How many cents had he in all?

6. There were 35 sheep in a lot, and 7 jumped out. How many remained?

7. What will be the cost of 6 dolls at 7 cents each?

8. How many pounds of rice can be bought for 42 cents, at 7 cents a pound?

9. How many are $\frac{1}{7}$ of 35 books?

10. Add at sight:

4	6	5	7	7
7	7	7	3	7

11. Subtract at sight:

10	13	12	14	7
7	7	7	7	5

12. Multiply at sight:

7	6	7	8	7
4	7	6	7	5

13. Divide at sight:

$7)\underline{14}$ $7)\underline{28}$ $7)\underline{49}$ $7)\underline{35}$

LESSON XLV.

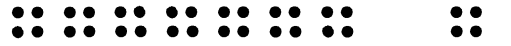
1. How many are $7+1$? $6+2$? $5+3$? $4+4$?
2. How many are $8+5$? $8+6$? $8+7$? $8+8$? $8+3$? $8+2$? $8+1$? $8+4$?
3. How many are $2+8$? $3+8$? $5+8$? $7+8$? $6+8$? $4+8$?
4. Count 16 by 2's. By 4's. By 8's.
5. How many are 16 and 8 more? $16+8=?$
6. Count 24 by 2's. By 4's. By 6's. By 8's.
7. Count 32 by 2's. By 4's. By 8's.
8. How many 4's in 32? How many 8's?
9. How many are 32 and 8 more? $32+8=?$
10. Count 40 by 2's. By 4's. By 5's. By 8's.
11. How many 8's are there in 40? How many 5's?
12. How many are $8+8+8$? $8+8+8+8+8$?
13. How many are $10-8$? $12-8$? $11-8$? $14-8$? $16-8$? $8-8$? $9-8$? $15-8$? $13-8$? $40-8$? $32-8$? $24-8$?
14. How many are 8×2? 8×4? 8×3? 8×5? 8×7? 8×6? 8×8? 3×8? 7×8? 6×8? 5×8?
15. How many are $8\div8$? $24\div8$? $32\div8$? $40\div8$? $64\div8$? $16\div8$? $56\div8$? $48\div8$?

LESSON XLVI.

1. If 8 equal boxes are filled with 32 pounds of tea, into how many *equal parts* are the 32 pounds divided?

When a number is divided into 8 equal parts, one of the parts is called **one eighth** of the number.

2. How many pounds are *one eighth* of 32 pounds?



$32\div8=4$, or *one eighth* of 32 is **4**.

3. How many are one eighth of 16 men? Of 24 men?

4. If 40 boys sit upon 8 benches, with an equal number on each, how many boys sit upon 1 bench?

One eighth is written thus, $\frac{1}{8}$. $\frac{1}{8}$ of 48 is $48 \div 8 = 6$.

$\frac{1}{8}$ of $8 = 1$ $\frac{1}{8}$ of $24 = 3$ $\frac{1}{8}$ of $40 = 5$ $\frac{1}{8}$ of $56 = 7$
$\frac{1}{8}$ of $16 = 2$ $\frac{1}{8}$ of $32 = 4$ $\frac{1}{8}$ of $48 = 6$ $\frac{1}{8}$ of $64 = 8$

5. How many are $\frac{1}{8}$ of 24 miles? Of 16? Of 32? Of 48? Of 56? Of 40?

6. Robert found 16 ripe peaches under a tree; he ate 3 and gave away 5. How many had he left?

7. George had 24 cents, which was 8 more than Ella had. How many cents had Ella?

8. At 8 cents each, what will 6 glasses cost?

9. At 8 cents a yard, how many yards of ribbon can be bought for 48 cents?

10. How many are $\frac{1}{8}$ of 48 houses?

11. Write the proper numbers in place of (?):

$16 + 8 = ?$ $16 - 8 = ?$ $7 \times 8 = ?$ $56 \div 8 = ?$

12. Add at sight:

8	8	5	3	4	7	8	7	5	4	4	5
6	7	8	8	8	8	5	1	8	0	6	8
						3	4	6	8	7	6
—	—	—	—	—	—	—	—	—	—	—	—

13. Subtract at sight:

10	12	16	8	24	14	9	8	8	8	32	10
8	8	8	3	8	8	8	4	2	5	8	2
—	—	—	—	—	—	—	—	—	—	—	—

14. Multiply at sight:

8	5	8	8	9	8	8	6	2	10	8	8
3	8	6	7	8	4	8	8	8	8	5	9
—	—	—	—	—	—	—	—	—	—	—	—

15. Divide at sight:

$8)\underline{56}$ $8)\underline{40}$ $8)\underline{64}$ $8)\underline{72}$ $8)\underline{48}$ $8)\underline{16}$ $8)\underline{24}$ $8)\underline{32}$

LESSON XLVII.

1. How many are $8+1$? $7+2$? $6+3$? $5+4$?

2. How many are $9+7$? $9+8$? $9+9$? $9+3$? $9+4$? $9+5$? $9+6$? $9+2$?

3. How many are $3+9$? $5+9$? $2+9$? $4+9$? $6+9$? $8+9$? $7+9$? $9+9$?

4. Count 18 by 2's. By 3's. By 6's. By 9's.

5. Count 27 by 3's. Count 27 by 9's.

6. How many 3's are there in 27? How many 9's in 27? How many 5's, and how many over?

7. How many are 27 less 9? 18 less 9? 9 less 9?

8. Count 36 by 2's. By 3's. By 4's. By 6's. By 9's.

9. How many are $27-9$? $18-9$? $16-9$? $14-9$? $36-9$? $13-9$? $15-9$? $11-9$? $12-9$? $10-9$? $17-9$? $63-9$? $54-9$? $45-9$?

10. Count 45 by 3's. By 5's. By 9's.

11. How many are 45 and 9 more? $45+9=?$

12. How many are 54 and 9 more? $54+9=?$

13. Count 63 by 3's. By 7's. By 9's.

14. How many 7's are there in 63? How many 9's?

15. How many are $9+9+9$? $9+9+9+9$?

16. How many are 9×2? 9×4? 9×3? 9×5? 9×7? 9×6? 9×9? 9×8?

17. How many are 4×9? 6×9? 5×9? 7×9? 9×9? 8×9? 3×9? 2×9?

18. How many are $18\div9$? $27\div9$? $36\div9$? $45\div9$? $54\div9$? $63\div9$? $81\div9$? $72\div9$?

19. Copy and add or count the following:

9	9	9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9	9	9
9	8	7	6	5	4	3	2	1	0
—	—	—	—	—	—	—	—	—	—

LESSON XLVIII.

1. If 45 trees are set in 9 rows, with an equal number in each row, into how many *equal parts* are they divided?

When a number is divided into 9 equal parts, one of the parts is called **one ninth** of the number.

2. How many trees are *one ninth* of 45 trees?

$45 \div 9 = 5$, or *one ninth* of 45 is 5.

3. How many rods are one ninth of 9 rods? Of 18 rods? Of 36 rods? Of 45 rods? Of 63 rods?

4. If 27 bushels of grain are put into 9 bags of equal size, what part of the 27 bushels will 1 bag contain?

One ninth is written thus, $\frac{1}{9}$. $\frac{1}{9}$ of 54 is $54 \div 9 = 6$.

$\frac{1}{9}$ of $9 = 1$	$\frac{1}{9}$ of $36 = 4$	$\frac{1}{9}$ of $63 = 7$
$\frac{1}{9}$ of $18 = 2$	$\frac{1}{9}$ of $45 = 5$	$\frac{1}{9}$ of $72 = 8$
$\frac{1}{9}$ of $27 = 3$	$\frac{1}{9}$ of $54 = 6$	$\frac{1}{9}$ of $81 = 9$

5. How many are $\frac{1}{9}$ of 27 gallons? Of 36? Of 63?

6. I have 18 nuts and 9 raisins. How many more nuts than raisins have I?

7. At 6 cents each, how much will 9 pencils cost?

8. At 9 cents a pound, how many pounds of sugar can be bought for 45 cents? For 54 cents? For 27 cents?

9. How many are one ninth of 36 books? Of 54?

10. Add at sight:

9	9	7	9	9
5	6	9	8	9

11. Subtract at sight:

10	12	18	27	15
9	9	9	9	9

12. Multiply at sight:

9	8	9	7
5	9	6	9

13. Divide at sight:

$9)\underline{27}$ $9)\underline{54}$ $9)\underline{63}$ $9)\underline{81}$

LESSON XLIX.

1. How many are $9+1$? $7+3$? $6+4$? $5+5$? $8+2$? $10+5$? $10+7$? $8+10$? $10+2$? $10+3$? $4+10$? $10+6$? $9+10$? $10+11$? $10+10$?

2. Count by 2's to 20. By 4's. By 5's. By 10's.

3. How many 2's in 20? How many 4's? 5's? 10's?

4. How many are 20 and 10? 30 and 10? 40 and 10?

5. How many 10's in 30? In 40? In 50? In 60?

6. Count 40 by 2's. By 4's. By 5's. By 8's. By 10's.

7. Count by 10's from 1 to 91. From 2 to 92. From 3 to 93. From 4 to 94. From 5 to 95. From 6 to 96. From 7 to 97. From 8 to 98. From 0 to 100.

8. How many are $14-10$? $17-10$? $20-10$? $21-10$? $19-10$? $18-10$? $16-10$? $13-10$? $15-10$? $12-10$? $10-10$? $11-10$? $100-10$? $90-10$? $70-10$? $40-10$?

9. How many are 10×3? 5×10? 10×6? 7×10? 2×10? 10×4? 9×10? 10×10? 8×10?

10. How many are $10\div10$? $100\div10$? $60\div10$? $30\div10$? $20\div10$? $40\div10$? $50\div10$? $70\div10$? $80\div10$? $90\div10$?

When a number is divided into 10 equal parts, one of the parts is called **one tenth** of the number.

11. How many barrels are *one tenth* of 70 barrels?

$70\div10=7$, or *one tenth* of 70 is 7.

12. How many sheep are one tenth of 80 sheep?

One tenth is written thus, $\frac{1}{10}$. $\frac{1}{10}$ of 90 is $90\div10=9$.

$\frac{1}{10}$ of $10=1$ $\frac{1}{10}$ of $40=4$ $\frac{1}{10}$ of $70=7$

$\frac{1}{10}$ of $20=2$ $\frac{1}{10}$ of $50=5$ $\frac{1}{10}$ of $80=8$

$\frac{1}{10}$ of $30=3$ $\frac{1}{10}$ of $60=6$ $\frac{1}{10}$ of $90=9$

13. How many are $\frac{1}{10}$ of 90 cups? Of 60? Of 100?

LESSON L.

1. How many are $10+1$? $8+3$? $9+2$? $7+4$? $6+5$? $11+4$? $11+6$? $11+9$? $11+8$? $7+11$? $10+11$? $11+3$? $5+11$? $11+11$? $2+11$?

2. Count by 2's to 22. How many 2's are there in 22? How many 11's in 22?

3. How many 11's in 33? In 44? In 55? In 66?

4. Count 44 by 2's. By 4's. By 11's.

5. How many are $66+11$? $77+11$? $88+11$?

6. How many are $22-11$? $20-11$? $17-11$? $18-11$? $14-11$? $10-11$? $21-11$? $19-11$? $16-11$? $15-11$? $13-11$? $11-11$? $12-11$? $99-11$? $88-11$? $77-11$? $66-11$? $55-11$?

7. How many are 11×2? 11×3? 11×4? 5×11? 11×6? 11×7? 11×8? 9×11? 11×10? 11×11?

8. How many are $11\div11$? $22\div11$? $55\div11$? $33\div11$? $44\div11$? $66\div11$? $77\div11$? $99\div11$? $88\div11$? $110\div11$? $121\div11$?

When a number is divided into 11 equal parts, one of the parts is called **one eleventh.**

9. How many boys are *one eleventh* of 44 boys?

$44\div11=4$, or *one eleventh* of 44 is 4.

10. How many days are one eleventh of 55 days?

One eleventh is written thus, $\frac{1}{11}$. $\frac{1}{11}$ of 66 is $66\div11=6$.

$\frac{1}{11}$ of $11=1$ $\frac{1}{11}$ of $55=5$ $\frac{1}{11}$ of $99=9$

$\frac{1}{11}$ of $22=2$ $\frac{1}{11}$ of $66=6$ $\frac{1}{11}$ of $110=10$

$\frac{1}{11}$ of $33=3$ $\frac{1}{11}$ of $77=7$ $\frac{1}{11}$ of $121=11$

$\frac{1}{11}$ of $44=4$ $\frac{1}{11}$ of $88=8$

11. How many are $\frac{1}{11}$ of 88 nuts? Of 66? Of 77? Of 33?

LESSON LI.

1. How many are $11+1$? $8+4$? $9+3$? $7+5$?

2. How many are $12+4$? $3+12$? $12+5$? $6+12$? $12+7$? $8+12$? $9+12$? $10+12$? $12+11$? $12+12$?

3. Count by 2's to 24. By 4's. By 6's. By 8's. By 12's.

4. How many are $24-12$? $23-12$? $21-12$? $22-12$? $13-12$? $15-12$? $14-12$? $17-12$? $16-12$? $19-12$? $18-12$? $20-12$? $84-12$?

5. How many are 3×12? 4×12? 12×6? 5×12? 12×1? 7×12? 12×8? 9×12? 10×12? 12×11? 12×12? 12×2?

6. What is $\frac{1}{2}$ of 12? $\frac{1}{3}$ of 12? $\frac{1}{4}$ of 12? $\frac{1}{6}$ of 12?

7. How many 12's in 24? In 36? In 48? In 60?

8. How many are $132\div12$? $144\div12$? $108\div12$? $120\div12$? $96\div12$? $72\div12$? $84\div12$? $60\div12$? $48\div12$? $36\div12$? $24\div12$? $12\div12$?

9. Count 60 by 3's. By 5's. By 6's. By 10's. By 12's.

10. How many are $24+12$? $36+12$? $48+12$? $96-12$? $84-12$?

When a number is divided into 12 equal parts, one of the parts is called **one twelfth.**

11. How many eggs are *one twelfth* of 60 eggs?

$60\div12=5$, or *one twelfth* of 60 is 5.

One twelfth is written $\frac{1}{12}$. $\frac{1}{12}$ of 72 is $72\div12=6$.

$\frac{1}{12}$ of $12=1$	$\frac{1}{12}$ of $60=5$	$\frac{1}{12}$ of $108=9$
$\frac{1}{12}$ of $24=2$	$\frac{1}{12}$ of $72=6$	$\frac{1}{12}$ of $120=10$
$\frac{1}{12}$ of $36=3$	$\frac{1}{12}$ of $84=7$	$\frac{1}{12}$ of $132=11$
$\frac{1}{12}$ of $48=4$	$\frac{1}{12}$ of $96=8$	$\frac{1}{12}$ of $144=12$

12. How many are $\frac{1}{12}$ of 132 jars? Of 144? Of 84?

RUDIMENTS OF ARITHMETIC.

DEFINITIONS.*

1. **Quantity** is any thing that can be increased, diminished, or measured; as distance, space, weight, motion, time.

2. A **Unit** is a single thing, or a number of single things regarded as a whole.

3. **Number** is a term signifying how many things are to be considered. A **Number** is a unit, or a collection of units.

4. An **Abstract Number** is a number used without reference to any particular object.

Thus, 3, 24, 756 are abstract numbers.

5. A **Concrete Number** is a number applied to an object.

Thus, 21 hours, 4 cents, 230 miles are concrete numbers.

6. A **Simple Number** is either an abstract number, or a concrete number of but one denomination.

Thus, 48, 52 pounds, 36 days are simple numbers.

* Some of these definitions are repeated elsewhere under the subjects to which they specially apply. They are here grouped together for convenience.

7. A **Compound Number** is a concrete number expressed in two or more denominations.

Thus, 4 bushels, 3 pecks; 8 rods, 4 yards; 2 feet, 3 inches are compound numbers.

8. An **Integral Number,** or **Integer,** is a number which expresses whole things.

Thus, 5, 12 dollars, 17 men are integral numbers.

9. A **Fractional Number,** or **Fraction,** is a number which expresses equal parts of a whole thing or quantity.

Thus, $\frac{1}{2}$, $\frac{3}{4}$ of a pound, $\frac{5}{12}$ of a bushel are fractional numbers.

10. Like Numbers have the same kind of unit, or express the same kind of quantity.

Thus, 74 and 16 are like numbers; so are 74 pounds and 16 pounds; also, 4 weeks, 3 days; and 16 minutes, 20 seconds, both being used to express units of time.

11. Unlike Numbers have different kinds of units, or are used to express different kinds of quantity.

Thus, 36 miles, and 15 days; 5 hours, 36 minutes; and 7 bushels, 3 pecks are unlike numbers.

12. Arithmetic is the *science* of numbers, and the *art* of computation. It treats of numbers and their uses.

13. The **Five Fundamental Operations** of Arithmetic are, *Notation* and *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

NOTATION AND NUMERATION.

14. Notation is a method of *writing* or expressing numbers by means of figures.

15. Numeration is a method of *reading* numbers expressed by figures.

16. Two systems of Notation are in general use — the *Roman* and the *Arabic*.

THE ROMAN NOTATION.

17. This system of notation employs seven capital letters to express numbers.

LETTERS.	I	V	X	L	C	D	M
VALUES.	One	Five	Ten	Fifty	One hundred	Five hundred	One thousand

18. The Roman notation is founded upon the following principles:

1. Repeating a letter repeats its value.

Thus, II represents two, XX twenty, CCC three hundred.

2. When a letter is placed *after* another of greater value, its value is to be *added to* that of the greater.

Thus, XI represents eleven, LX sixty, DC six hundred.

3. When a letter is placed *before* another of greater value, its value is to be *taken from* that of the greater.

Thus, IX represents nine, XL forty, CD four hundred.

4. When a letter is placed *between* two letters, each of greater value, its value is to be *taken from* the *sum* of the other two.

Thus, XIV represents fourteen, XXIX twenty-nine, XCIV ninety-four.

TABLE OF ROMAN NOTATION.

I, One.	XII, Twelve.	XL, Forty.
II, Two.	XIII, Thirteen.	L, Fifty.
III, Three.	XIV, Fourteen.	LX, Sixty.
IV, Four.	XV, Fifteen.	LXX, Seventy.
V, Five.	XVI, Sixteen.	LXXX, Eighty.
VI, Six.	XVII, Seventeen.	XC, Ninety.
VII, Seven.	XVIII, Eighteen.	C, One hundred.
VIII, Eight.	XIX, Nineteen.	CC, Two hundred.
IX, Nine.	XX, Twenty.	D, Five hundred.
X, Ten.	XXI, Twenty-one.	DC, Six hundred.
XI, Eleven.	XXX, Thirty.	M, One thousand.

Express by the Roman notation:

1. Fourteen.
2. Nineteen.
3. Twenty-four.
4. Thirty-nine.
5. Forty-six.
6. Fifty-one.
7. Eighty-eight.
8. Seventy-three.
9. Ninety-five.
10. One hundred one.

THE ARABIC NOTATION.*

19. This system of notation employs ten characters or figures to express numbers.

FIGURES.	0	1	2	3	4	5	6	7	8	9
NAMES.	Naught	One	Two	Three	Four	Five	Six	Seven	Eight	Nine

20. The first character is called *naught*, *cipher*, or *zero*, because it has no value of its own. The other nine characters are called *significant figures*, because each has a value of its own.

* A fuller presentation of the numbers from 1 to 100 will be found in the Introductory Exercises.

21. As we have no single character to represent ten, we express it by writing the unit, 1, at the left of the cipher, 0.

Thus, 10.

22. When a number is expressed by two figures, the right hand figure is called *units*, and the left hand figure *tens*.

The greatest number that can be expressed by *two* figures is 99.

23. We express one hundred by writing the unit, 1, at the left hand of two ciphers.

Thus, 100.

In like manner we write two hundred, 200, three hundred, 300, etc., to nine hundred, 900.

24. When a number is expressed by three figures, the right hand figure is called *units*, the second figure *tens*, and the left hand figure *hundreds*.

The greatest number that can be expressed by *three* figures is 999.

Express the following numbers by figures:

1. One hundred twenty.
2. Four hundred eighty-three.
3. Seven hundred sixteen.
4. Nine hundred.
5. Two hundred ninety.
6. Eight hundred nine.

25. We express one thousand by writing the unit, 1, at the left hand of three ciphers.

Thus, 1000.

In the same manner we write two thousand, three thousand, etc., to nine thousand; thus,

One thousand	Two thousand	Three thousand	Four thousand	Five thousand
1000	2000	3000	4000	5000

Six thousand	Seven thousand	Eight thousand	Nine thousand
6000	7000	8000	9000

26. When a number is expressed by four figures, the places, commencing at the right hand, are *units*, *tens*, *hundreds*, *thousands*.

To express hundreds, tens, and units with thousands, we write in each place the figure indicating the number we wish to express in that place.

Thus, to write four thousand two hundred and sixty-nine, we write 4 in the place of thousands, 2 in the place of hundreds, 6 in the place of tens, 9 in the place of units; thus,

Thousands.	Hundreds.	Tens.	Units.
4	2	6	9

The greatest number that can be expressed by *four* figures is 9999.

Express the following numbers by figures:

1. One thousand two hundred.
2. Five thousand one hundred sixty.
3. Three thousand seven hundred forty-one.
4. Eight thousand fifty-six.
5. Two thousand ninety.
6. Seven thousand nine.
7. One thousand one.
8. Nine thousand four hundred twenty-seven.
9. Four thousand thirty-five.
10. One thousand nine hundred four.

Read the following numbers:

11.	76	128	405	910	116	3414	1025.
12.	2100	5047	7009	4670	3997	1001	5639.

27. Next to thousands come *tens* of thousands, and next to these come *hundreds* of thousands, as tens and hundreds come in their order after units.

Write the following numbers in figures:

1. Twenty thousand.
2. Forty-seven thousand.
3. Eighteen thousand, one hundred.
4. Twelve thousand, three hundred fifty.
5. Thirty-nine thousand, five hundred twenty-two.
6. Fifteen thousand, two hundred six.
7. Eleven thousand, twenty-four.
8. Forty thousand ten.
9. Sixty thousand, six hundred.
10. Eight hundred forty thousand, three hundred.

Read the following numbers:

11.	5006	12304	96071	5470	203410.
12.	36741	400560	13061	49000	100010.

For convenience in reading large numbers, we may point them off, by commas, into periods of three figures each, counting from the right hand or unit figure. This pointing enables us to read the hundreds, tens, and units in each period with facility as seen in the following table:

NUMERATION TABLE.

PERIODS.	3d.			2d.			1st.		
NAME.	Millions			Thousands			Units		
ORDERS OF UNITS.	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
NUMBER.	1	3	6,	4	0	8,	0	6	0

28. Figures occupying different places in a number, as units, tens, hundreds, etc., are said to express different orders of units.

29. In *numerating*, or expressing numbers verbally, the various orders of units have the following names:

Orders.	Names.
1st order is called	Units.
2d order " "	Tens.
3d order " "	Hundreds.
4th order " "	Thousands.
5th order " "	Tens of thousands.
6th order " "	Hundreds of thousands.
7th order " "	Millions.
8th order " "	Tens of millions.
9th order " "	Hundreds of millions.

Write and read the following numbers:

1. One unit of the third order, two of the second, five of the first. *Ans.* 125; read, *one hundred twenty-five.*

2. Two units of the 5th order, four of the 4th, five of the 2d, 6 of the 1st.

Ans. 24056; read, *twenty-four thousand fifty-six.*

3. Seven units of the 4th order, five of the 3d, three of the 2d, eight of the 1st.

4. Two units of the 6th order, nine of the 5th, four of the 3d, one of the 1st, seven of the 2d.

5. Three units of the 6th order, four of the 2d.

6. Nine units of the 5th order, seven of the 4th, nine of the 1st.

7. Four units of the 7th order, two of the 6th, one of the 3d, five of the 2d.

8. Eight units of the 7th order, three of the 6th, five of the 5th, two of the 3d, eight of the 1st.

30. Since the number expressed by any figure depends upon the place it occupies, it follows that figures have two values, *Simple* and *Local.*

31. The **Simple Value** of a figure is its value when taken alone;

Thus, 4, 7, 2.

32. The **Local Value** of a figure is its value when used with another figure or figures in the same number.

Thus, in 325, the local value of the 3 is 300, of the 2 is 20, and of the 5 is 5 units.

When a figure occupies units' place, its simple and local values are the same.

33. The leading principles upon which the Arabic notation is founded are embraced in the following laws:

General Laws.

I. *All numbers are expressed by applying the ten figures to the different orders of units.*

II. *The different orders of units increase from right to left, and decrease from left to right, in a tenfold ratio.*

III. *Every removal of a figure one place to the left, increases its local value tenfold; and every removal of a figure one place to the right, diminishes its local value tenfold.*

From this analysis of the principles of Notation and Numeration, we derive the following rules:

Rule for Notation. I. *Beginning at the left hand, write the figures belonging to the highest period.*

II. *Write the hundreds, tens, and units, of each successive period in their order, placing a cipher wherever an order of units is omitted.*

Rule for Numeration. I. *Separate the number into periods of three figures each, beginning at the right hand.*

II. *Beginning at the left hand, read each period separately, and give the name to each period, except the last, or period of units.*

34. Until the pupil can write numbers readily, it may be well for him to write several periods of ciphers, point them off, over each period write its name, and then write the given numbers underneath, in their appropriate places.

Thus,

OOO, OOO, OOO.

Millions, Thousands, Units.

EXERCISES IN NOTATION AND NUMERATION.

Express the following numbers by figures:

1. Four hundred thirty-six.
2. Seven thousand, one hundred sixty-four.
3. Twenty-six thousand, twenty-six.
4. Fourteen thousand, two hundred eighty.
5. One hundred seventy-six thousand.
6. Four hundred fifty thousand, thirty-nine.

7. Four hundred eighty-three million, eight hundred sixteen thousand, one hundred forty-nine.

8. Ninety-five million.

9. Nine hundred thousand, ninety.

10. Ten million, ten thousand, ten hundred, ten.

Point off, numerate, and read the following numbers:

11. 8240.	15. 111111.	19. 370005.
12. 400900.	16. 57468139.	20. 706342.
13. 308.	17. 5628.	21. 8429526.
14. 60720.	18. 11111111.	22. 111111111.

23. Write seven million thirty-six.

24. Write five hundred sixty-three thousand, four.

25. Write one million, ninety-six thousand.

26. A certain number contains 3 units of the seventh order, 6 of the fifth, 4 of the fourth, 1 of the third, 5 of the second, and 2 of the first; what is the number?

27. What orders of units are contained in the number 290648?

ADDITION.

35. Addition is the process of uniting *several* numbers of the same kind into *one* equivalent number.

36. The **Sum** or **Amount** is the result obtained.

ORAL EXERCISES.

1. A farmer paid 6 dollars for a straw-cutter, and 9 dollars for a plow. What did he pay for both?

SOLUTION. — He paid the sum of 6 dollars and 9 dollars, which is 15 dollars.

2. John gave 4 apples to James, 8 to Henry, and 9 to Fred. How many did he give to all?

3. I gave 7 dollars for a barrel of flour, 5 dollars for a hundred-weight of sugar, and 6 dollars for a tub of butter. What did I give for the whole?

4. I have two pear trees; last year one tree produced 12 bushels of pears, and the other 11 bushels. How many bushels did both produce?

5. A man bought 4 cords of wood for 12 dollars, and 7 bushels of corn for 5 dollars. What did he pay for both?

6. James gave 11 cents for a slate, and had 8 cents left. How many cents had he at first?

7. A lady paid 5 dollars for a bonnet, 10 dollars for a shawl, and had 7 dollars left. How much money had she at first?

8. In a shop there are 8 men, 9 boys, and 6 girls at work. How many persons are at work in the shop?

9. Robert bought a quire of paper for 12 cents, and a slate for 13 cents; he gave 10 cents to his sister. How much money did he pay out in all?

10. A man bought 4 bushels of wheat for 7 dollars, 18 bushels of corn for 11 dollars, and 2 cords of wood for 5 dollars. What did he pay for the whole?

11. A farmer has 6 cows in one yard, 9 in another, and as many in the third yard as in both the others. How many cows has he?

12. There were 7 desks on the first row of a school-room, 7 desks on the second row, and 6 on the third row. How many desks were there in the three rows?

13. Mary is 8 years old, Jane is 3 years older than Mary, and Anna is 4 years older than Jane. How old is Jane? How old is Anna?

14. A little boy had 9 cents in his bank, his aunt gave him 5 cents, and his uncle gave him 10 cents. How much did he then have in the bank?

15. George has 8 marbles in his pocket, 4 in his right hand, and 5 in his left hand. How many marbles has he altogether?

37. The **Sign of Addition** is the perpendicular cross, +, called *plus*. It shows that the numbers connected by it are to be *added*.

Thus, 3 + 4 + 7, read 3 *plus* 4 *plus* 7, or 3 and 4 and 7.

38. The **Sign of Equality** is two short, parallel, horizontal lines =. It shows that the numbers, or combination of numbers, connected by it are *equal*.

Thus, 4 + 8 = 9 + 3, read the sum of 4 and 8 is *equal* to the sum of 9 and 3.

Promiscuous Addition Table.

2 + 5 =	5 + = 13	7 + 9 =	4 + = 10
6 + 2 =	3 + = 10	6 + 5 =	7 + = 10
2 + 4 =	6 + = 10	3 + 6 =	2 + = 10
8 + 9 =	7 + = 13	4 + 4 =	5 + = 14
9 + 4 =	6 + = 14	7 + 8 =	8 + = 16
4 + 7 =	9 + = 15	9 + 3 =	6 + = 13
8 + 6 =	8 + = 11	4 + 3 =	5 + = 10
6 + 3 =	8 + = 17	3 + 8 =	9 + = 16
7 + 2 =	5 + = 12	5 + 6 =	9 + = 18
3 + 9 =	2 + = 7	5 + 8 =	6 + = 15
4 + 5 =	6 + = 8	3 + 7 =	7 + = 14
9 + 8 =	2 + = 6	6 + 4 =	3 + = 7
8 + 5 =	8 + = 19	7 + 6 =	8 + = 15
4 + 9 =	9 + = 13	6 + 8 =	4 + = 12
5 + 4 =	4 + = 11	9 + 5 =	9 + = 11
2 + 7 =	8 + = 14	8 + 3 =	5 + = 8
7 + 5 =	6 + = 9	9 + 6 =	6 + = 12
5 + 2 =	7 + = 9	5 + 7 =	7 + = 11
6 + 9 =	7 + = 16	4 + 6 =	3 + = 12
7 + 7 =	6 + = 11	7 + 3 =	4 + = 9
3 + 4 =	3 + = 9	2 + 8 =	9 + = 17
8 + 7 =	4 + = 8	5 + 9 =	8 + = 13
4 + 8 =	7 + = 15	8 + 8 =	4 + = 13
9 + 2 =	9 + = 12	6 + 7 =	8 + = 12
5 + 3 =	5 + = 9	5 + 5 =	2 + = 9
6 + 6 =	3 + = 11	9 + 7 =	7 + = 12
7 + 4 =	5 + = 11	9 + 9 =	5 + = 7

EXAMPLES.

39. When the amount of each column is less than 10.

1. A drover bought three flocks of sheep. The first contained 232, the second 422, and the third 245 sheep. How many sheep did he buy in all?

OPERATION.

	hunds.	tens.	units.
	2	3	2
	4	2	2
	2	4	5
Amount,	8	9	9

SOLUTION. — We arrange the numbers so that units of like order stand in the same column. Then we add the columns separately, for convenience commencing at the right hand, and we write each result under the column added. Thus, we have 5, 7, 9, the sum of the units; 4, 6, 9, the sum of the tens; 2, 6, 8, the sum of the hundreds. The entire amount is 8 hundreds 9 tens and 9 units, or 899.

2.	3.	4.	5.
403	164	510	234
271	321	176	324
124	510	203	140
Ans. 798			

6.	7.	8.	9.
1234	2041	3102	4100
2405	3216	2253	1523
5140	1500	4014	2041
Ans. 8779			

10. What is the sum of 421, 305, and 5162?
11. What is the sum of 3121, 436, and 2002?
12. What is the sum of 4325, 452, and 112?
13. What is the sum of 5125, 643, and 2331?
14. What is the sum of 6658, 2320, and 1021?

40. When the amount of any column equals or exceeds 10.

1. A merchant pays 397 dollars for freight, 476 dollars for a clerk, and 873 dollars for rent of a store. What is the amount of his expenses?

OPERATION.

$$\begin{array}{r} 397 \\ 476 \\ 873 \\ \hline 1746 \end{array}$$

SOLUTION. — We arrange the numbers so that units of like order stand in the same column. Then we add the first, or right hand column, and the sum is 16 units, or 1 ten and 6 units; writing the 6 units under the column of units, we add the 1 ten to the column of tens, and the sum is 24 tens, or 2 hundreds and 4 tens; writing the 4 tens under the column of tens, we add the 2 hundreds to the column of hundreds, and the sum is 17 hundreds, or 1 thousand and 7 hundreds; writing the 7 hundreds under the column of hundreds, and the 1 in thousands' place, we have the entire sum, 1746.

1. In adding, always pronounce the partial results without naming the *figures* separately. Thus, in the operation given for illustration, say 3, 9, 16; 8, 15, 24; 10, 14, 17.

2. When the sum of any column is greater than 9, the process of adding the tens to the next column is called *carrying*.

41. From the preceding examples and illustrations we deduce the following rule:

Rule. I. *Write the numbers to be added so that all the units of the same order stand in the same column; that is, units under units, tens under tens, etc.*

II. *Commencing at units, add each column separately, and write the sum underneath, if it is less than ten.*

III. *If the sum of any column is ten or more than ten, write the unit figure only, and add the ten or tens to the next column.*

IV. *Write the entire sum of the last column.*

Proof. *Begin with the right hand or unit column, and add the figures in each column in an opposite direction to that in which they were first added; if the two results agree, the work is probably correct.*

2. Inches.	3. Feet.	4. Pounds.	5. Yards.	6. Miles.
142	325	75	407	1270
325	46	276	96	342
476	674	508	2584	79
943	1045	859	3087	1691

7.	8.	9.	10.	11.
842	376	426	713	4761
396	407	397	86	374
472	862	450	345	83
205	94	294	60	19

12. What is the sum of $912 + 342 + 187 + 46$?

Ans. 1487.

13. What is the sum of 56 feet, 450 feet, and 680 feet?

Ans. 1186 feet.

14. What is the sum of 1942 dollars and 685 dollars?

Ans. 2627 dollars.

15. A man paid 375 dollars for a span of horses, 160 dollars for a carriage, and 87 dollars for a harness. What did he pay for all? *Ans.* 622 dollars.

16. A man traveled 476 miles by railroad, 390 miles by steamboat, and 120 miles by stage. How many miles did he travel in all? *Ans.* 986 miles.

17. A carpenter built a house for 2464 dollars, a barn for 496 dollars, and out-houses for 309 dollars. What did he receive for building all?

18. A merchant bought at public auction 520 yards of broadcloth, 386 yards of muslin, 92 yards of flannel, and 156 yards of silk. How many yards did he buy in all?

19. A father divided his estate among his four sons, giving each $ 2087. What was the amount of his estate?

NOTE. — The sign $ placed before a number signifies *dollars*.

20. Three persons deposited money in a bank; the first put in $ 4780, the second $ 3042, and the third $ 407. How much did they all deposit?

21. Five men engage in business as partners, and each puts in $ 2375. What is the whole amount of capital invested? *Ans.* $ 11875.

22.	23.	24.	25.
765	347	630	4603
381	192	815	7106
976	763	456	972
315	410	307	385
169	507	960	64

Ans. 2606

26.	27.	28.
767346	374205	4076315
432761	108497	5632870
286109	643024	8219634
508763	879638	3827692

Ans. 2094979

29. $3720 + 647 + 190 + 82 = ?$ *Ans.* 4639.

30. $962 + 2161 + 500 + 75 = ?$ *Ans.* 3698.

31. $4170 + 1009 + 642 + 120 + 18 = ?$

32. $3000 + 47602 + 805 + 1266 + 76 = ?$

33. $69 + 4030 + 349 + 1384 + 72 + 400 = ?$

34. What is the sum of two thousand eight hundred fifty-six, twelve thousand eighty-four, seven hundred forty-two, and sixty-nine? *Ans.* 15751.

35. What is the amount of twenty thousand five hundred ten, six thousand nine hundred forty-four, and three thousand two hundred? *Ans.* 30654.

36. What is the sum of forty-seven thousand fifty, nine thousand one hundred six, fourteen hundred ninety-two, and five hundred twelve? *Ans.* 58160.

37. What is the sum of one hundred forty thousand three hundred thirty-four, seventy-nine thousand six hundred five, twenty-five hundred twenty-five, and three thousand sixty-nine? *Ans.* 225533.

38. What is the amount of five hundred thousand five hundred five, eighty-four thousand two hundred, fifteen thousand six hundred twenty, and seventeen hundred seventeen? *Ans.* 602042.

39. How many men are there in an army consisting of 26840 infantry, 6370 cavalry, 3250 dragoons, 750 artillery, and 320 miners? *Ans.* 37530.

40. A merchant deposited $125 in bank on Monday, $91 on Tuesday, $164 on Wednesday, $200 on Thursday, $196 on Friday, and $73 on Saturday. How much did he deposit during the week?

41. By selling a farm for $3586, $684 were lost. What did the farm cost?

42. I was born in 1840, when will I be 63 years old?

43. A man willed his estate to his wife, two sons, and three daughters. To his daughters he gave $1565 apiece, to his sons $3560 each, and to his wife $4720. What was his estate worth? *Ans.* $16535.

44. A man engaging in trade, gained $450 the first year, $684 the second year, and as much the third year as

he gained during the first and second years. What was his whole gain? *Ans.* $2268.

45. I bought three village lots for $12570, and sold them so as to gain $745 on each lot. For how much did I sell them? *Ans.* $14805.

46. A has $3240, B has $5672, and C has $1000 more than A and B together. How many dollars have all?

Ans. $18824.

47. A man was 32 years old when his son was born. How old will he be when his son is 36 years old?

Ans. 68 years.

48. The Old Testament contains 39 books, 929 chapters, 23214 verses, 592439 words, and 2728100 letters; the New Testament contains 37 books, 269 chapters, 7959 verses, 181153 words, and 838380 letters. What is the total number of each in the Bible?

Ans. 76 books, 1198 chapters, 31173 verses, 773592 words, and 3566480 letters.

49. The number of immigrants landed in New York in three successive years, was 418422, 338784, and 364086. What was the total number landed in the three years?

Ans. 1121292.

50. According to the census of 1890, the population of New York was 1515301; of Chicago, 1099850; of Philadelphia, 1046964; of Brooklyn, 806343; of St. Louis, 451770; of Boston, 448477; of Baltimore, 434439. What was the total population of these cities?

Ans. 5803144.

51. In three years, the United States exported molasses to the value of $154630, $108003, $115893. What was the value of the molasses exported in the three years?

Ans. $378526.

52. During the same years, the United States exported tobacco valued at $1829207, $1458553, and $2410224.

What was the total value of the tobacco exported in those years? *Ans.* $5697984.

53. How many miles from the southern extremity of Lake Michigan to the Gulf of St. Lawrence, passing through Lake Michigan, 330 miles; Lake Huron, 260 miles; St. Clair River, 24 miles; Lake St. Clair, 20 miles; Detroit River, 23 miles; Lake Erie, 260 miles; Niagara River, 34 miles; Lake Ontario, 180 miles; St. Lawrence River, 750 miles? *Ans.* 1881 miles.

54. In the year 1890 there were in operation in the New England States, 6897 miles of railroad; in New York, 7745 miles; in Pennsylvania, 8700 miles; in Ohio, 7987; in Virginia, 3367; in Illinois, 10129; and in Georgia, 4592. What was the whole number of miles in operation in all these States? *Ans.* 49417.

55. The number of pieces of silver coin made at the United States Mints in the year 1891 were as follows: 36232802 standard dollars, 165275 half dollars, 780475 quarter dollars, and 17614621 dimes. What was the total number of pieces coined? *Ans.* 54793173.

56.	57.	58.	59.
344	843	1186	81988
388	738	513	380167
613	237	740	108424
803	218	1820	193686
825	347	955	144225
412	288	736	112558
322	483	810	107481
886	753	511	176826
620	834	1179	145851
5213	4741	8450	1451206

60.	61.	62.	63.
35938	47197	12380	456568
49172	63956	98795	754712
56546	85678	23442	567346
82564	35495	87639	543678
69789	16457	91758	342766
47321	94667	19347	768345
77563	76463	81731	563875
83563	34698	29342	547427
54973	17179	75659	945956
38137	93965	35446	165675
54246	81367	98237	756431
95864	29787	12845	354747
48135	79826	87677	543864
37975	31275	23444	567456
48467	59689	39878	621367

64.	65.	66.	67.
768856	576654	987654	9873785
674387	678456	123456	1239564
978874	754543	876864	7591074
567678	786567	234246	3517569
568594	964432	765183	8598674
639678	699678	345927	2513756
669657	978321	654678	3454210
594886	678789	456432	7656754
695756	564673	345719	5467856
789568	895437	765391	5645781
689689	569128	673123	7893344
638786	678982	437987	3216677
675968	869771	566789	4569911
958789	668339	544321	6543344
769896	956234	891389	9576677
153674	195876	219721	1539900
331767	957412	625247	6662233
355989	573375	431321	4235566
11522492	13046667	9945448	99796675

SUBTRACTION.

42. Subtraction is taking away part of a number, or finding the difference between two numbers of the same unit value.

43. The **Remainder** or **Difference** is the result obtained.

ORAL EXERCISES.

1. A grocer having 20 boxes of lemons sold 12 boxes. How many boxes had he left?

SOLUTION. — Twelve boxes taken from 20 boxes leave 8 boxes; he had 8 boxes left.

2. If a man earns 12 dollars a week, and spends 7 for provisions, how many dollars has he left?

3. If I borrow 15 dollars, and pay 9 dollars, how many dollars remain unpaid?

4. John had 11 marbles, and lost 5 of them. How many had he left?

5. From a cistern containing 22 barrels of water, 9 barrels leaked out. How many barrels remained?

6. In a school there are 24 boys and 12 girls. How many more boys than girls are there?

SOLUTION. — The difference between 12 and 24 is 12; there are 12 more boys than girls.

7. From a piece of cloth containing 17 yards, 8 yards were cut. How many yards remained?

8. Cora is 23 years old, and her brother is 10 years younger. How old is her brother?

44. The **Minuend** is the number from which another is to be subtracted.

45. The **Subtrahend** is the number to be subtracted.

46. The **Sign of Subtraction** is a short horizontal line — called *minus*. When placed between two numbers, it denotes that the one after it is to be taken from the one before it.

Thus, 8 − 6 = 2, is read 8 *minus* 6 equals 2, and shows that 6, the *subtrahend*, taken from 8, the *minuend*, equals 2, the *remainder*.

PROMISCUOUS SUBTRACTION TABLE.

14 − 5 =	11 − = 4	14 − 6 =	9 − = 1
9 − 3 =	10 − = 3	15 − 8 =	16 − = 7
10 − 9 =	12 − = 5	11 − 5 =	11 − = 2
7 − 6 =	13 − = 7	10 − 3 =	8 − = 5
12 − 7 =	12 − = 8	13 − 3 =	14 − = 9
12 − 9 =	16 − = 9	11 − 9 =	13 − = 9
10 − 5 =	13 − = 6	12 − 6 =	9 − = 7
11 − 6 =	12 − = 4	10 − 8 =	10 − = 2
9 − 8 =	16 − = 8	11 − 4 =	14 − = 5
16 − 7 =	15 − = 9	14 − 8 =	9 − = 6
11 − 2 =	11 − = 7	12 − 5 =	10 − = 4
8 − 5 =	12 − = 9	13 − 7 =	14 − = 7
14 − 9 =	15 − = 6	12 − 8 =	11 − = 3
13 − 9 =	18 − = 9	16 − 9 =	13 − = 5
9 − 7 =	10 − = 6	13 − 6 =	17 − = 9
10 − 2 =	13 − = 4	12 − 4 =	14 − = 8
15 − 7 =	14 − = 6	16 − 8 =	15 − = 7
17 − 8 =	15 − = 8	15 − 9 =	17 − = 8
10 − 4 =	11 − = 5	11 − 7 =	10 − = 9
14 − 7 =	10 − = 8	12 − 3 =	7 − = 6
11 − 3 =	13 − = 8	15 − 6 =	12 − = 7
13 − 5 =	11 − = 9	18 − 9 =	12 − = 6

EXAMPLES.

47. When no figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 574 take 323.

OPERATION.	
Minuend,	574
Subtrahend,	323
Remainder,	251

SOLUTION. — The less number is written under the greater, with units placed under units, tens under tens, etc., and a line is drawn beneath.

Then, we begin at the right hand, and subtract separately each figure of the subtrahend from the figure above it in the minuend. Thus 3 taken from 4 leaves 1, which is the difference of the units; 2 taken from 7 leaves 5, the difference of the tens; 3 taken from 5 leaves 2, the difference of the hundreds. Hence, we have for the whole difference, 2 hundreds 5 tens and 1 unit, or 251.

	2.	3.	4.	5.
Minuend,	876	349	637	508
Subtrahend,	435	212	431	104
Remainder,	441	137	206	404

	6.	7.	8.	9.
From	987	753	438	695
Take	647	502	421	535
	340	251	17	160

	10.	11.	12.	13.
From	7642	8730	2369	9786
Take	3211	6430	2104	3126
	4431	2300	265	6660

14. From 4376 take 1254. *Ans.* 3122.

15. From 70342 take 50130. *Ans.* 20212.

	Remainders.
16. From 137647 take 16215.	121432.
17. Subtract 217356 from 719568.	502212.
18. 437615 − 213502 = ?	224113.
19. 732740 − 11520 = ?	721220.
20. 2042674 − 32142 = ?	
21. 8461203 − 7161003 = ?	
22. Subtract 32014 from 86325.	54311.

23. From one hundred eighty-three thousand four hundred sixty, take fifty-two thousand one hundred fifty. *Ans.* 131310.

24. A man bought a piece of property for $7634, and sold the same for $3132. What did he lose? *Ans.* $4502.

25. A merchant sold goods to the amount of $41763, and by so doing gained $11521. What did the goods cost him? *Ans.* $30242.

26. A drover bought 3245 sheep, and sold 1249 of them. How many sheep had he left?

27. A general before commencing a battle had 18765 men in his army; after the battle he had only 8530. How many men did he lose? *Ans.* 10235.

28. Two persons bought a block of buildings for $69524; one paid $47321. How much did the other pay? *Ans.* $22203.

29. If a man's annual income is $13460, and his expenses are $3340, what does he save? *Ans.* $10120.

30. From three thousand two hundred seventy-six, take two thousand one hundred forty-three.

31. Mr. Black inherited a fortune of $19435, of which he gave $15000 to his son. How much had he left? *Ans.* $4435.

32. A man who had $3989256 lost $1765000 in business. How much had he left? *Ans.* $2224256.

48. When any figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 846 take 359.

OPERATION.

	hunds.	tens.	units.
Minuend,	8	4	6
Subtrahend,	3	5	9
Remainder,	4	8	7

SOLUTION. — Since we cannot take 9 units from 6 units, we add 10 units to 6 units, making 16 units; and 9 units taken from 16 units leave 7 units. But as we have added 10 units or 1 ten to the minuend, we shall have a remainder 1 ten too large, to avoid which, we add 1 ten to the 5 tens in the subtrahend, making 6 tens. We cannot take 6 tens from 4 tens; so we add 10 tens to 4, making 14 tens; 6 tens taken from 14 tens leave 8 tens. Now, having added 10 tens, or 1 hundred, to the minuend, we shall have a remainder 1 hundred too large, unless we add 1 hundred to the 3 hundreds in the subtrahend, making 4 hundreds; 4 hundreds taken from 8 hundreds leave 4 hundreds, and we have for the total remainder, 487.

The process of adding 10 to the minuend is sometimes called *borrowing* 10; and that of adding 1 to the next figure of the subtrahend, *carrying* one.

49. From the preceding example and illustration we have the following general rule:

Rule. I. *Write the less number under the greater, placing units of the same order in the same column.*

II. *Beginning at the right hand, take each figure of the subtrahend from the figure in the minuend and write the result underneath.*

III. *If any figure in the subtrahend is greater than the corresponding figure in the minuend add 10 to the minuend figure before subtracting, and then add 1 to the next left-hand figure of the subtrahend.*

Proof. I. *Add the remainder to the subtrahend; the sum will be equal to the minuend. Or,*

II. *Subtract the remainder from the minuend; the difference will be equal to the subtrahend.*

	2.	3.	4.	5.
Minuend,	753	6731	3248	90361
Subtrahend,	469	2452	1863	6284
Remainder,	284	4279	1385	84077

	6.	7.	8.	9.
	Miles.	Bushels.	Dollars.	Feet.
	3146	19472	45268	24760
	2529	14681	24873	3478
	617	4791	20395	21282

	10.	11.	12.	13.
	Rods.	Days.	Acres.	Gallons.
	40307	14605	23617	980076
	38421	8341	14309	94087
	1886	6264	9308	885989

	14.	15.	16.	17.
	Men.	Sheep.	Barrels.	Tons.
	17380	282731	80014	941000
	3417	90756	43190	5007
	13963	191975	36824	935993

	18.	19.	20.
	3077097	3000001	1970000
	1829164	2199077	1361111
	1247933	800924	608889

21.	22.	23.
6000000	8000800	103810040
999999	457776	91300397
5000001	7543024	12509643

24. $234100 - 9970 = ?$ *Ans.* 224130.

25. $4000320 - 20142 = ?$

26. $14601896 - 764059 = ?$

27. From 4716359 take 2740714. *Ans.* 1975645.

28. From 7867564 take 2948675. *Ans.* 4918889.

29. From 7788996 take 849842. *Ans.* 6939154.

30. From 1073563 take 182000. *Ans.* 891563.

31. From 1111111 take 111112. *Ans.* 999999.

32. Subtract 1234509 from 8643587. *Ans.* 7409078.

33. Subtract 1000 from 1100000. *Ans.* 1099000.

34. Subtract 100701 from 846587.

35. Subtract 432986702100 from 539864298670.

36. Subtract 29176807982 from 86543298765.

37. A speculator bought wild lands for $10580, and sold them for $7642. How much did he lose?

Ans. $2938.

38. Napoleon the Great was born in 1769, and died in 1821. How old was he at the time of his death?

Ans. 52 years.

39. Gunpowder was invented in 1330, and printing in 1440. How many years between the two? *Ans.* 110.

40. George Washington was born in 1732, and died in 1799. How old was he at his death? *Ans.* 67 years.

41. The first newspaper published in America was issued at Boston in 1704. How long was that before the death of Benjamin Franklin, which occurred in 1790?

Ans. 86 years.

42. The first steamboat in the United States, built by Robert Fulton, in 1807, made a trip from New York to

Albany in 33 hours. How many years from that time to the visit of the Great Eastern to this country in 1860?

Ans. 53 years.

43. Queen Victoria was born in 1819. What was her age in 1892? *Ans.* 73 years.

44. The United States contains 3668167 square miles, and British North America contains 3777550 square miles. How many square miles does the latter country exceed the former? *Ans.* 109383.

EXAMPLES COMBINING ADDITION AND SUBTRACTION.

1. A farmer having 450 sheep, sold 124 at one time, and 96 at another. How many had he left? *Ans.* 230.

2. If a man's income is $175 a month, and he pays $25 for rent, $44 for provisions, and $18 for other expenses, how much will he have left? *Ans.* $88.

3. A man gave his note for $3245. He paid at one time $780, and at another $484. How much remained unpaid? *Ans.* $1981.

4. A man paid $140 for a horse and $165 for a carriage. He afterwards sold them both for $300. Did he gain or lose, and how much? *Ans.* Lost $5.

5. A flour merchant having 700 barrels of flour on hand, sold 278 barrels to one man, and 142 to another. How many barrels had he left? *Ans.* 280 barrels.

6. Three men bought a farm for $9840. The first paid $2672, the second paid $3089, and the third, the remainder. What did the third pay? *Ans.* $4079.

7. A man bought a house for $1500, and having expended $315 for repairs, sold it for $2000. What was his gain? *Ans.* $185.

8. Henry Jones owns property to the amount of $36748, of which he has invested in real estate $12850,

in personal property $ 9086, and the remainder he has in bank. How much has he in bank? *Ans.* $ 14812.

9. A grocer bought 275 pounds of butter of one farmer, and 318 pounds of another; he afterwards sold 210 pounds to one customer, and 97 to another. How many pounds had he left? *Ans.* 286 pounds.

10. A man deposited in bank $ 10476; he drew out at one time $ 2356, at another $ 1242, and at another $ 737. How much had he remaining in bank? *Ans.* $ 6141.

11. Borrowed of my neighbor at one time $ 680, at another time $ 910, and at another time $ 218. Having paid him $ 1309, how much do I still owe him? *Ans.* $ 499.

12. A man bought 3 lots; for the first he paid $ 2480, for the second $ 3137, and for the third as much as for the other two; he afterwards sold them for $ 15000. What was his gain? *Ans.* $ 3766.

13. A farmer raised 1864 bushels of wheat, and 1129 bushels of corn. Having sold 1340 bushels of wheat, and 1000 bushels of corn, how many bushels of each has he remaining? *Ans.* 524 bushels, and 129 bushels.

14. A man worth $ 25800, bequeathed his estate so that each of his two sons should have $ 9400, and his daughter the remainder. What was the daughter's portion?

MULTIPLICATION.

50. Multiplication is the process of taking one of two given numbers as many times as there are units in the other.

51. The **Product** is the result obtained.

ORAL EXERCISES.

1. At 9 cents a pound, what will 7 pounds of sugar cost?

SOLUTION. — Seven times 9 cents are 63 cents. Seven pounds of sugar will cost 63 cents.

2. At $6 a week, what will 8 weeks' board cost?

3. At $7 a yard, what will 9 yards of cloth cost?

4. If Robert can earn $10 in one month, how much can he earn in 4 months? In 9 months? In 11 months?

5. What will be the cost of 12 pounds of coffee, at 9 cents a pound?

6. At $5 a ton, what will 9 tons of coal cost?

7. At $4 a yard, what will 8 yards of cloth cost?

8. If a pair of boots costs $5, what will be the cost of 3 pairs? Of 6 pairs? Of 7 pairs? Of 11 pairs?

9. Since 12 inches make a foot, how many inches in 3 feet? In 5 feet? In 7 feet? In 12 feet?

10. At 5 cents a quart, what will 6 quarts of milk cost? 10 quarts? 11 quarts?

11. If a man earns $8 in a week, how much can he earn in 6 weeks? In 7 weeks? In 8 weeks?

52. The **Multiplicand** is the number to be multiplied.

53. The **Multiplier** is the number which shows how many times the multiplicand is to be taken.

54. The **Factors** are the multiplicand and multiplier.

55. The **Sign of Multiplication** is the oblique cross, ×. It shows that the numbers connected by it are to be multiplied by one another.

Thus, $9 \times 6 = 54$, is read 9 *times* 6 equal 54.

PROMISCUOUS MULTIPLICATION TABLE.

$3 \times 8 =$	$7 \times \ = 49$	$2 \times 9 =$	$3 \times \ = 21$
$3 \times 9 =$	$4 \times \ = 8$	$6 \times 5 =$	$3 \times \ = 27$
$4 \times 8 =$	$9 \times \ = 81$	$4 \times 7 =$	$4 \times \ = 32$
$7 \times 5 =$	$4 \times \ = 12$	$9 \times 3 =$	$9 \times \ = 18$
$9 \times 4 =$	$6 \times \ = 54$	$5 \times 8 =$	$9 \times \ = 36$
$6 \times 3 =$	$2 \times \ = 12$	$5 \times 7 =$	$6 \times \ = 18$
$4 \times 9 =$	$8 \times \ = 40$	$9 \times 5 =$	$4 \times \ = 36$
$5 \times 9 =$	$4 \times \ = 16$	$6 \times 7 =$	$5 \times \ = 25$
$7 \times 6 =$	$9 \times \ = 72$	$8 \times 3 =$	$7 \times \ = 42$
$3 \times 7 =$	$2 \times \ = 4$	$7 \times 7 =$	$8 \times \ = 56$
$8 \times 9 =$	$5 \times \ = 45$	$4 \times 2 =$	$5 \times \ = 20$
$6 \times 8 =$	$8 \times \ = 72$	$9 \times 9 =$	$3 \times \ = 15$
$5 \times 6 =$	$3 \times \ = 9$	$4 \times 3 =$	$3 \times \ = 12$
$7 \times 3 =$	$2 \times \ = 6$	$6 \times 9 =$	$8 \times \ = 48$
$6 \times 4 =$	$7 \times \ = 28$	$2 \times 6 =$	$7 \times \ = 56$
$9 \times 7 =$	$3 \times \ = 18$	$8 \times 5 =$	$5 \times \ = 15$
$4 \times 5 =$	$6 \times \ = 60$	$4 \times 4 =$	$3 \times \ = 30$
$7 \times 4 =$	$4 \times \ = 20$	$9 \times 6 =$	$8 \times \ = 64$
$8 \times 7 =$	$2 \times \ = 18$	$2 \times 4 =$	$5 \times \ = 10$
$5 \times 4 =$	$6 \times \ = 30$	$4 \times 6 =$	$8 \times \ = 16$
$3 \times 5 =$	$4 \times \ = 28$	$9 \times 8 =$	$6 \times \ = 48$
$3 \times 4 =$	$9 \times \ = 27$	$3 \times 3 =$	$5 \times \ = 30$

1. Factors are producers, and the multiplicand and multiplier are called factors because they produce the product.

2. Multiplication is a short method of performing addition when the numbers to be added are equal.

EXAMPLES.

56. When the multiplier consists of one figure.

1. Multiply 374 by 6.

OPERATION.

	hunds.	tens.	units.
Multiplicand,	3	7	4
Multiplier,			6
Product,	2	2	4 4

SOLUTION. — In this example it is required to take 374 *six* times. If we take the units of each order 6 times, we shall take the entire number 6 times. Therefore, writing the multiplier under the unit figure of the multiplicand, we proceed as follows: 6 times 4 units are 24 units, which are 2 *tens* and 4 *units;* we write the 4 units in the product in units' place, and reserve the two tens to add to the next product; 6 times 7 tens are 42 tens, and the two tens reserved in the last product added, are 44 tens, which are 4 *hundreds* and 4 *tens;* we write the 4 tens in the product in tens' place, and reserve the 4 hundreds to add to the next product; 6 times 3 hundreds are 18 hundreds, and 4 hundreds added are 22 hundreds, which, being written in the product in the places of hundreds and thousands, the entire product is 2244.

57. The *unit value* of a number is not changed by repeating the number. As the multiplier always expresses *times*, the product must have the same unit value as the multiplicand. But, since the product of any two numbers will be the same, whichever factor is taken as a multiplier, either factor may be taken for the multiplier or multiplicand.

In multiplying, learn to pronounce the partial results, as in addition, without naming the numbers separately. Thus, in the last example, instead of saying 6 times 4 are 24, 6 times 7 are 42 and 2 to carry are 44, 6 times 3 are 18 and 4 to carry are 22; pronounce only the results, 24, 44, 22, performing the operations mentally. This will greatly facilitate the process of multiplying.

	2.	3.	4.	5.
Multiplicand,	842	625	718	937
Multiplier,	4	6	7	3
Product,	3368	3750	5026	2811

	6.	7.	8.	9.
	4328	5073	1869	3265
	8	5	4	9
	34624	25365	7476	29385

	10.	11.	12.	13.
	7186	9010	4079	6394
	3	7	6	8
	21558	63070	24474	51152

	14.	15.	16.
	340071	760892	1976230
	2	4	5
	680142	3043568	9881150

17. Multiply 473126 by 9. *Ans.* 4258134.

18. Multiply 789167 by 7. *Ans.* 5524169.

19. Multiply 231420 by 8. *Ans.* 1851360.

20. What will be the cost of 9380 barrels of flour, at $9 a barrel? *Ans.* $84420.

21. What will be the cost of 4738 tons of coal, at $4 a ton? *Ans.* $18952.

22. In one mile there are 5280 feet. How many feet are there in 8 miles? *Ans.* 42240 feet.

58. When the multiplier consists of two or more figures.

1. Multiply 746 by 23.

OPERATION.

Multiplicand, 746
Multiplier, 23

2238 3 { times the multiplicand.
1492 20 { times the multiplicand.
Product, 17158 23 { times the multiplicand.

SOLUTION. — Writing the multiplicand and multiplier as in **56**, we first multiply each figure in the multiplicand by the unit figure of the multiplier, precisely as in **56**. Then we multiply by the 2 tens as shown in the example. Adding the two partial products, we have for the entire product, 17158.

From the preceding example we deduce the following general rule:

Rule. I. *Write the multiplier under the multiplicand, placing units of the same order under one another.*

II. *Multiply the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in addition.*

III. *If there are partial products, add them, and their sum will be the product required.*

Proof. *Multiply the multiplier by the multiplicand, and if the product is the same as the first result, the work is probably correct.*

When the multiplier contains two or more figures, the several results obtained by multiplying by each figure are called *partial* products.

2.	3.	4.
34732	56784	34075
14	24	36
138928	227136	204450
34732	113568	102225
486248	1362816	1226700

5. Multiply 177242 by 19. *Ans.* 3367598.
6. Multiply 364111 by 56. *Ans.* 20390216.
7. Multiply 78540 by 95. *Ans.* 7461300.
8. Multiply 6555 by 39. *Ans.* 255645.
9. Multiply 76419 by 17. *Ans.* 1299123.
10. Multiply 26517 by 45. *Ans.* 1193265.
11. Multiply 108336 by 58. *Ans.* 6283488.
12. Multiply 209402 by 72. *Ans.* 15076944.
13. Multiply 342516 by 56. *Ans.* 19180896.
14. Multiply 764131 by 48. *Ans.* 36678288.

15. There are 52 weeks in a year. How many weeks are there in 1861 years? *Ans.* 96772 weeks.

16. An army of 5746 men having plundered a city, took so much money that each man received $37. How much money was taken? If the army in their haste lost $10000, how much did they have left?

17. If it costs $47346 to build one mile of railroad, what will it cost to build 76 miles at the same price per mile? *Ans.* $3598296.

18.	19.	20.
47696	560341	243042
144	1304	265
190784	2241364	1215210
190784	1681023	1458252
47696	560341	486084
6868224	730684664	64406130

21. Multiply 45678 by 333. *Ans.* 15210774.
22. Multiply 202842 by 342. *Ans.* 69371964.
23. Multiply 9636799 by 489. *Ans.* 4712394711.
24. Multiply 3064125 by 807. *Ans.* 2472748875.
25. Multiply 5610327 by 2034. *Ans.* 11411405118.
26. Multiply 1900731 by 4006. *Ans.* 7614328386.
27. A gentleman bought 307 horses, at the rate of $105 each. How much did he pay for them?
28. What will be the value of 976 shares of railroad stock, at $98 a share? *Ans.* $95648.
29. A man bought 48 building lots, at $1236 each. What did they all cost him? *Ans.* $59328.
30. How many yards of broadcloth are there in 487 pieces, if each piece contains 37 yards?

Ans. 18019 yards.

31. If it requires 135 tons of iron for one mile of railroad, how many tons will be required for 196 miles?

Ans. 26460 tons.

32. How many oranges in 356 boxes, if each box contains 264 oranges? *Ans.* 93984 oranges.
33. If it requires 6894 shingles for the roof of a house, how many shingles will be required for the roofs of 19 such houses?
34. $37896 \times 149 = ?$ *Ans.* 5646504.
35. $8566 \times 462 = ?$
36. $6793 \times 842 = ?$ *Ans.* 5719706.
37. $674200 \times 2104 = ?$ *Ans.* 1418516800.
38. $15607 \times 3094 = ?$ *Ans.* 48288058.
39. $83209 \times 4004 = ?$
40. Multiply 31416 by 175.
41. Multiply 40930 by 779. *Ans.* 31884470.
42. Multiply 4567 by 9009. *Ans.* 41144103.
43. Multiply 7071 by 556. *Ans.* 3931476.
44. Multiply 291042 by 125. *Ans.* 36380250.

45. Multiply 54001 by 5009.

46. Multiply twelve thousand thirteen, by twelve hundred four. *Ans.* 14463652.

47. Multiply 670306 by 1234.

48. Multiply 9000000 by 3045.

49. Multiply 543210 by 98765.

50. Multiply 1498344 by 498344.

51. Multiply 89765432 by 234567.

52. Multiply 1928374 by 56498.

53. Multiply two thousand twelve by one thousand six hundred forty-nine. *Ans.* 3317788.

54. Multiply thirty-seven thousand seven hundred ninety-six, by four hundred eight.

55. Multiply one million seven hundred seventy-five thousand eight hundred fifty-seven, by eight thousand two hundred forty-one. *Ans.* 14634837537.

56. Multiply one million two hundred forty-six thousand eight hundred fifty-three, by nine thousand seven. *Ans.* 11230404971.

57. What will be the cost of 309 acres of land at $2450 an acre?

58. What will be the cost of building 128 miles of railroad, at $6375 per mile? *Ans.* $816000.

59. A crop of cotton was put up in 126 bales, each bale containing 572 pounds. What was the weight of the entire crop? *Ans.* 72072 pounds.

60. Two towns, 243 miles apart, are to be connected by a railroad, at a cost of $39760 a mile. What will be the entire cost of the road? *Ans.* $9661680.

61. If each of 246 acres of land produces 105 bushels, how much will they all produce? *Ans.* 25830 bushels.

62. If a garrison of soldiers consumes 5789 pounds of bread a day, how much will it consume in 287 days? *Ans.* 1661443 pounds.

CONTRACTIONS.

59. A **Composite Number** is one that may be produced by multiplying two or more numbers by one another.

Thus, 18 is a composite number, since $6 \times 3 = 18$; or, $9 \times 2 = 18$; or, $3 \times 3 \times 2 = 18$.

60. The **Component Factors** of a number are the several numbers which, multiplied by one another, produce the given number.

Thus, the component factors of 20 are 10 and 2, $(10 \times 2 = 20)$; or, 4 and 5, $(4 \times 5 = 20)$; or, 2 and 2 and 5, $(2 \times 2 \times 5 = 20)$.

EXAMPLES.

61. When the multiplier is a composite number.

1. What will 32 horses cost, at 174 dollars apiece?

OPERATION.

Multiplicand,	174	cost of 1 horse.
1st factor,	4	
	696	cost of 4 horses.
2d factor,	8	
Product,	5568	cost of 32 horses.

SOLUTION. — The factors of 32 are 4 and 8. If we multiply the cost of 1 horse by 4, we obtain the cost of 4 horses; and by multiplying the cost of 4 horses by 8, we obtain the cost of 8 times 4 horses, or 32 horses.

Hence the following rule:

Rule. I. *Separate the composite number into two or more factors.*

II. *Multiply the multiplicand by one of these factors, and the product by another, and so on until all the factors have been used; the last product will be the product required.*

The product of any number of factors will be the same in whatever order they are multiplied. Thus, $4 \times 3 \times 5 = 60$, and $5 \times 4 \times 3 = 60$.

2. Multiply 521 by 16 or 4×4. *Ans.* 8336.

3. Multiply 10709 by 36 or 6×6. *Ans.* 385524.

4. Multiply 21700 by 27 or 3×9. *Ans.* 585900.

5. Multiply 783473 by 42 or 6×7. *Ans.* 32905866.

6. Multiply 764131 by 48 or 6×8. *Ans.* 36678288.

7. Multiply 40567 by 96 or 8×12. *Ans.* 3894432.

8. Multiply 182642 by 120 or $4 \times 5 \times 6$.

Ans. 21917040.

9. Multiply 20704 by 84 or $3 \times 4 \times 7$.

Ans. 1739136.

10. Multiply 564120 by 140 or $4 \times 5 \times 7$.

Ans. 78976800.

11. What will 56 acres of land cost, at $147 an acre?

Ans. $8232.

12. What will 75 yoke of cattle cost, at $184 a yoke?

Ans. $13800.

13. If a ship sails 380 miles a day, how far will she sail in 45 days? *Ans.* 17100 miles.

14. What is the value of 3426 pounds of butter, at 18 cents a pound? *Ans.* $616.68.

NOTE. — 61668 cents equals 616 dollars and 68 cents, and is usually written $616.68.

15. What will be the cost of 125 horses, at $208 each?

Ans. $26000.

16. What is the value of 1342 acres of land, at $28 an acre?

17. What will be the cost of 28 pieces of broadcloth, each piece containing 42 yards, at $4 a yard?

Ans. $4704.

18. What will be the cost of 16 sacks of coffee, each sack containing 75 pounds, at 9 cents a pound?

Ans. $108.

62. When the multiplier is 100, 1000, etc.

If we annex a cipher to the multiplicand, each figure is removed *one* place toward the left, and consequently the value of the whole number is increased tenfold. If two ciphers are annexed, each figure is removed *two* places toward the left, and the value of the number is increased one hundredfold; and every additional cipher increases the value tenfold.

Hence the following rule:

Rule. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier; the number so formed is the product required.*

1. Multiply 246 by 10. *Ans.* 2460.
2. Multiply 97 by 100. *Ans.* 9700.
3. Multiply 1476 by 1000. *Ans.* 1476000.
4. Multiply 7361 by 10000. *Ans.* 73610000.
5. At $47 an acre, what will 10 acres of land cost?
6. What will be the cost of 100 horses, at $95 a head? *Ans.* $9500.
7. What will be the cost of 1000 fruit trees, at 18 cents apiece? *Ans.* $180.
8. If one acre of land produces 28 bushels of wheat in a year, how many bushels will 100 such acres produce in a year? *Ans.* 2800.
9. If a man saves $386 a year, what will he save in 10 years? *Ans.* $3860.
10. If the freight on a barrel of flour from Chicago to New York is 47 cents, what will it be on 100000 barrels? *Ans.* $47000.
11. The owners of a piano factory sold 1000 pianos at $245 each. What did they receive for them?

Ans. $245000.

63. When there are ciphers at the right hand of one or both of the factors.

1. Multiply 7200 by 40.

OPERATION.	
Multiplicand,	7200
Multiplier,	40
Product,	288000

SOLUTION. — The multiplicand, factored, is equal to 72×100; the multiplier, factored, is equal to 4×10; and as these factors taken in any order will give the same product, we first multiply 72 by 4, then this product by 100 by annexing two ciphers, and this product by 10 by annexing one cipher.

Hence the following rule:

Rule. *Multiply the significant figures of the multiplicand by those of the multiplier, and to the product annex as many ciphers as there are ciphers on the right of both factors.*

	2.	3.	4.
Multiply	3900	1760	37200
By	8000	3500	730000
	31200000	880	1116
		528	2604
		6160000	27156000000

5. Multiply 7030 by 164000. *Ans.* 1152920000.
6. Multiply 27600 by 48000. *Ans.* 1324800000.
7. Multiply 403700 by 30200. *Ans.* 12191740000.
8. Multiply 99900 by 99900.
9. Multiply 345000 by 543000.
10. At $150 an acre, what will be the cost of 500 acres of land? *Ans.* $75000.
11. What will be the freight on 4000 barrels of flour, at 50 cents a barrel? *Ans.* $2000.
12. If there are 560 shingles in a bunch, how many shingles in 26000 bunches of the same size?

Ans. 14560000.

EXAMPLES COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. Bought 9 cords of wood at $3 a cord, and 15 tons of coal at $5 a ton. What was the cost of the wood and coal? *Ans.* $102.

2. A grocer bought 6 tubs of butter, each containing 64 pounds, at 14 cents a pound, and 4 cheeses, each weighing 42 pounds, at 8 cents a pound. What was the cost of the butter and cheese? *Ans.* $67.20.

3. If a clerk receives $540 a year salary, and pays $180 for board, $116 for clothing, $58 for books, and $75 for other expenses, how much will he have left at the close of the year? *Ans.* $111.

4. A farmer having $2150, bought 536 sheep at $2 a head, and 26 cows at $23 a head. How much money had he left? *Ans.* $480.

5. A man sold three horses; for the first he received $275, for the second $87 less than for the first, and for the third as much as for the other two. What did he receive for the third? *Ans.* $463.

6. Bought 76 hogs, each weighing 416 pounds, at 7 cents a pound, and sold the same at 9 cents a pound. What was gained? *Ans.* $632.32.

7. A man bought 14 cows at $26 each, 4 horses at $112 each, and 125 sheep at $3 each. He sold them all for $1237. Did he gain or lose, and how much?

Ans. Gained $50.

8. B has 174 sheep, C has three times as many lacking 98, and D has as many as B and C together. How many sheep has D? *Ans.* 598.

9. There are 36 tubs of butter, each weighing 108 pounds; the tubs which contain the butter each weigh 19 pounds. What is the weight of the butter without the tubs? *Ans.* 3204 pounds.

10. A man paid $2376 for building a house, and four times as much lacking $970 for his farm. What did he pay for both?

11. A merchant bought 9 hogsheads of sugar at $32 a hogshead, and sold it for $40 a hogshead. What was the gain? *Ans.* $72.

12. Bought 360 barrels of flour for $2340, and sold the same at $8 a barrel. What was gained by the bargain? *Ans.* $540.

13. A farmer sold 462 bushels of wheat at $2 a bushel, for which he received 75 barrels of flour at $9 a barrel, and the remainder in money. How much money did he receive? *Ans.* $249.

14. Two persons start from the same point, and travel in opposite directions; one travels at the rate of 28 miles a day, the other at the rate of 37 miles a day. How far apart will they be in 6 days? *Ans.* 390 miles.

15. If a man buys 40 acres of land at $35 an acre, and 56 acres at $29 an acre, and sells them for $32 an acre, what does he gain or lose? *Ans.* Gains $48.

16. In an orchard, 76 apple trees yield 18 bushels of apples each, and 27 others yield 21 bushels each. What are all these apples worth, at 30 cents a bushel?

Ans. $580.50.

17. A man bought two farms, one of 136 acres at $28 an acre, and another of 140 acres at $33 an acre. He paid at one time $4000, and at another time $1875. How much remained unpaid? *Ans.* $2553.

18. There are 60 seconds in a minute and 60 minutes in an hour. How many seconds are there in 24 hours?

Ans. 86400 seconds.

19. A horse dealer bought 15 horses at $345 each and 14 at $150 each. He sold them all for $10000. Did he gain or lose and how much? *Ans.* Gained $2725.

DIVISION.

64. **Division** is the process of finding *how many times* one number is contained in another of the same kind.

65. The **Dividend** is the number to be divided.

66. The **Divisor** is the number by which to divide.

67. The **Quotient** is the result obtained, and shows how many times the divisor is contained in the dividend.

ORAL EXERCISES.

1. How many barrels of flour, at $6 a barrel, can be bought for $30?

SOLUTION. — Six dollars is contained 5 times in $30. Five barrels of flour can be bought for $30.

2. How many oranges, at 4 cents apiece, can be bought for 28 cents?

3. How many tons of coal, at $5 a ton, can be bought for $35?

4. When lard is 7 cents a pound, how many pounds can be bought for 49 cents? For 63 cents? For 84 cents?

5. If a man travels 48 miles in 6 hours, how far does he travel in one hour?

6. At 3 cents apiece, how many lemons can be bought for 24 cents? For 30 cents? For 36 cents?

7. If you divide 55 cents equally among 5 children, how many cents do you give each?

68. The **Sign of Division** is a short horizontal line, with a point above and one below, $\div$. It shows that the number before it is to be divided by the number after it.

Thus, $20 \div 4 = 5$, is read, 20 *divided by* 4 is equal to 5.

Division is also expressed by writing the dividend *above* and the divisor *below* a short horizontal line;

Thus, $\frac{12}{3} = 4$, shows that 12 *divided by* 3 equals 4.

PROMISCUOUS DIVISION TABLE.

$36 \div 6 =$	$16 \div \quad = 2$	$\frac{63}{9} =$	$\frac{12}{\quad} = 3$
$42 \div 7 =$	$32 \div \quad = 4$	$\frac{12}{6} =$	$\frac{45}{\quad} = 5$
$81 \div 9 =$	$24 \div \quad = 6$	$\frac{28}{7} =$	$\frac{42}{\quad} = 6$
$35 \div 5 =$	$72 \div \quad = 9$	$\frac{16}{4} =$	$\frac{56}{\quad} = 8$
$72 \div 8 =$	$10 \div \quad = 5$	$\frac{49}{7} =$	$\frac{63}{\quad} = 7$
$27 \div 9 =$	$8 \div \quad = 4$	$\frac{12}{3} =$	$\frac{27}{\quad} = 3$
$20 \div 4 =$	$20 \div \quad = 5$	$\frac{64}{8} =$	$\frac{21}{\quad} = 7$
$54 \div 6 =$	$10 \div \quad = 2$	$\frac{40}{8} =$	$\frac{16}{\quad} = 8$
$32 \div 8 =$	$63 \div \quad = 9$	$\frac{28}{4} =$	$\frac{12}{\quad} = 4$
$45 \div 5 =$	$12 \div \quad = 6$	$\frac{14}{2} =$	$\frac{35}{\quad} = 7$
$42 \div 6 =$	$28 \div \quad = 7$	$\frac{48}{6} =$	$\frac{12}{\quad} = 5$
$56 \div 8 =$	$16 \div \quad = 4$	$\frac{45}{9} =$	$\frac{14}{\quad} = 7$
$63 \div 7 =$	$49 \div \quad = 7$	$\frac{48}{8} =$	$\frac{24}{\quad} = 4$
$27 \div 3 =$	$36 \div \quad = 4$	$\frac{56}{7} =$	$\frac{30}{\quad} = 5$
$21 \div 7 =$	$64 \div \quad = 8$	$\frac{21}{3} =$	$\frac{36}{\quad} = 9$
$16 \div 8 =$	$40 \div \quad = 8$	$\frac{24}{8} =$	$\frac{30}{\quad} = 6$
$12 \div 4 =$	$28 \div \quad = 4$	$\frac{16}{2} =$	$\frac{36}{\quad} = 6$
$35 \div 7 =$	$32 \div \quad = 8$	$\frac{32}{4} =$	$\frac{42}{\quad} = 7$
$10 \div 5 =$	$48 \div \quad = 6$	$\frac{24}{6} =$	$\frac{81}{\quad} = 9$
$14 \div 7 =$	$45 \div \quad = 9$	$\frac{72}{9} =$	$\frac{35}{\quad} = 5$
$24 \div 4 =$	$48 \div \quad = 8$	$\frac{36}{4} =$	$\frac{72}{\quad} = 8$
$30 \div 5 =$	$56 \div \quad = 7$	$\frac{8}{4} =$	$\frac{27}{\quad} = 9$
$36 \div 9 =$	$21 \div \quad = 3$	$\frac{20}{5} =$	$\frac{20}{\quad} = 4$
$30 \div 6 =$	$54 \div \quad = 6$	$\frac{10}{2} =$	$\frac{24}{\quad} = 3$
$24 \div 3 =$	$8 \div \quad = 2$	$\frac{40}{5} =$	$\frac{24}{\quad} = 8$

EXAMPLES.

69. When the divisor consists of but one figure.

1. How many times is 4 contained in 848?

OPERATION.

Dividend,
Divisor, 4)848
Quotient, 212

SOLUTION. — After writing the divisor on the left of the dividend, with a line between them, we begin at the left hand to divide: 4 is contained in 8, 2 times, and as 8 in the dividend is hundreds, the 2 in the quotient must be hundreds; therefore, we write 2 in hundreds' place under the figure divided. 4 is contained in 4, 1 time, and since 4 denotes tens, we write 1 under it in tens' place. 4 is contained in 8, 2 times, and since 8 is units, we write 2 in units' place under it, and we have 212 for the entire quotient.

	2.	3.	4.
Divisor.	3)936 Dividend.	2)4862	4)48844
	312 Quotient.	2431	12211

5. Divide 9963 by 3. *Ans.* 3321.
6. Divide 5555 by 5. *Ans.* 1111.
7. Divide 68242 by 2. *Ans.* 34121.
8. Divide 66666 by 6.

When the divisor is not contained in the *first* figure of the dividend, find how many times it is contained in the *first two* figures.

9. How many times is 4 contained in 2884?

OPERATION.

4)2884
721

SOLUTION. — Since we cannot divide 2 by 4, we find how many times 4 is contained in 28, which is 7 times, and since the 28 represents hundreds, we write the 7 in hundreds' place; then 4 is contained in 8, 2 times, which we write in tens' place under the figure divided; and 4 is contained in 4, 1 time, which we write in units' place in the quotient, and we have the entire quotient, 721.

10.	11.	12.
3)2469	5)3055	2)148624
823	611	74312

13. Divide 4266 by 6. *Ans.* 711.
14. Divide 36488 by 4. *Ans.* 9122.
15. Divide 72999 by 9. *Ans.* 8111.
16. Divide 21777 by 7.

After obtaining the first figure of the quotient, if the divisor is not contained in any figure of the dividend, place a cipher in the quotient, and *prefix* this figure to the next one of the dividend.

To *prefix* means to place *before*, or at the *left hand*.

17. How many times is 6 contained in 1824?

OPERATION.
6)1824
304

SOLUTION. — We begin as in the last examples. 6 is contained in 18, 3 times, which we place in hundreds' place in the quotient; then 6 is contained in 2 no times, so we place a cipher (0) in tens' place in the quotient. Adding the 2 tens to the 4 units, we have 24; 6 is contained in 24, 4 times, which we write in units' place, and we have 304 for the entire quotient.

18.	19.	20.
4)3228	7)28357	3)912246
807	4051	304082

21. Divide 40525 by 5. *Ans.* 8105.
22. Divide 36426 by 6. *Ans.* 6071.
23. Divide 184210 by 2. *Ans.* 92105.
24. Divide 85688 by 8. *Ans.* 10711.
25. Divide 273615 by 3. *Ans.* 91205.
26. Divide 16484 by 4. *Ans.* 4121.
27. Divide 25555 by 5. *Ans.* 5111.

After dividing any figure of the dividend, if there is a remainder, prefix it mentally to the next figure of the dividend, and then divide this number as before.

28. How many times is 4 contained in 943?

OPERATION.

4)943

235 . . . 3 Rem.

SOLUTION. — Here 4 is contained in 9 2 times, and there is a remainder of 1, which we prefix mentally to the next figure, 4; 4 is contained in 14, 3 times, with a remainder of 2, which we prefix to 3; 4 is contained in 23, 5 times, with a remainder of 3. This 3 which is left after performing the last division should be divided by the divisor 4; and we *indicate* the division by placing the divisor under it; thus, $\frac{3}{4}$. The entire quotient is written $235\frac{3}{4}$, which may be read, two hundred thirty-five and *three divided by four*, or, two hundred thirty-five and a *remainder of three*, or two hundred thirty-five and three fourths.

When the process of dividing is performed mentally, and the results only are written, as in the preceding examples, the operation is termed *Short Division*.

From the foregoing examples and illustrations, we deduce the following rule:

Rule. I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Beginning at the left hand, divide each figure of the dividend by the divisor, and write the result under the dividend.*

III. *When there is a remainder after dividing any figure, regard it as prefixed to the figure of the next lower order in the dividend, and divide as before.*

IV. *Should any figure or part of the dividend be less than the divisor, write a cipher in the quotient, and prefix the number to the figure of the next lower order in the dividend, and divide as before.*

V. *When there is a remainder after dividing the last figure, place it over the divisor at the right hand of the quotient.*

Proof. *Multiply the quotient by the divisor, and to the product add the remainder, if any; when the result is equal to the dividend, the work is correct.*

1. This method of proof depends on the fact that division is the reverse of multiplication. The *dividend* corresponds to the *product*, the divisor to one of the *factors*, and the *quotient* to the *other factor*.

2. In multiplication the two factors are given, to find the product: in division the product and one of the factors are given, to find the other factor.

29. Divide 8430 by 6.

OPERATION.		PROOF.
Divisor. 6)8430 Dividend.		1405 Quotient.
1405 Quotient.		6 Divisor.
		8430 Dividend.

30.	31.	32.
5)730490	7)510384	8)6003424
146098	72912	750428

	Quotients.
33. Divide 87647 by 7.	12521.
34. Divide 94328 by 8.	11791.
35. Divide 43272 by 9.	4808.
36. Divide 377424 by 6.	62904.
37. Divide 975216 by 8.	121902.
38. Divide 46375028 by 7.	6625004.
39. Divide 4763025 by 9.	529225.
40. Divide 42005607 by 7.	6000801.
41. Divide 72000450 by 9.	8000050.
42. Divide 97440643 by 8.	$12180080\frac{3}{8}$.
43. Divide 65706313 by 9.	$7300701\frac{4}{9}$.
44. Divide 3627089 by 6.	$604514\frac{5}{6}$.
45. Divide 4704091 by 7.	672013.

46. Divide $16344 equally among 6 men. What will each man receive? *Ans.* $2724.

47. How many barrels of flour, at $7 a barrel, can be bought for $87605? *Ans.* 12515 barrels.

48. There are 7 days in one week. How many weeks in 23044 days? *Ans.* 3292 weeks.

49. If 5 bushels of wheat make 1 barrel of flour, how many barrels of flour can be made from 314670 bushels? *Ans.* 62934 barrels.

50. By reading 9 pages a day, how many days will be required to read a book which contains 1161 pages? *Ans.* 129 days.

51. At $4 a yard, how many yards of broadcloth can be bought for $1372? *Ans.* 343 yards.

52. If a stage goes at the rate of 8 miles an hour, how long will it take to go 1560 miles? *Ans.* 195 hours.

53. There are 3 feet in 1 yard. How many yards in 206175 feet? *Ans.* 68725 yards.

54. Five partners share equally the loss of a ship and cargo, valued at $760315. What is each one's share of the loss? *Ans.* $152063.

55. If a township of 64000 acres is divided equally among 8 persons, how many acres will each receive? *Ans.* 8000 acres.

56. A miller wishes to put 36312 bushels of grain into 6 bins of equal size. How many bushels must each bin contain? *Ans.* 6052 bushels.

57. How many steps of 3 feet each would a man take in walking a mile, or 5280 feet? *Ans.* 1760 steps.

58. A gentleman left his estate, worth $36105, to be shared equally by his wife and 4 children. What did each receive? *Ans.* $7221.

59. There are 3 feet in one yard. How many yards are there in 1689 feet? *Ans.* 563.

70. When the divisor consists of two or more figures.

To illustrate more clearly the method of operation, we will first take an example usually performed by Short Division.

1. How many times is 4 contained in 1504?

OPERATION.

```
4)1504(376
  12
  ---
  30
  28
  --
  24
  24
  --
```

SOLUTION. — *First.* We find how many times the divisor, 4, is contained in 15, the first partial dividend, which is 3 times, with a remainder. We place this quotient figure at the right hand of the dividend, with a line between them. *Second.* To find the remainder we multiply the divisor, 4, by this quotient figure, 3, and place the product, 12, under the figures divided. Subtracting the product from the figures divided, there is a remainder of 3. *Third.* Annexing the next figure of the dividend to the right hand of the remainder, we have 30, the second *partial* dividend. 4 is contained in 30, 7 times, with a remainder. Placing the 7 at the right hand of the last quotient figure, and multiplying the divisor by it, we place the product, 28, under the figures last divided, and subtract as before. To the remainder, 2, we annex the next figure, 4, of the given dividend, and we have 24 for the third *partial* dividend. Then 4 is contained in 24, 6 times. Multiplying and subtracting as before, nothing remains, and we have for the entire quotient 376.

When the whole process of division is written out as above, the operation is termed *Long Division.* The *principle*, however, is the *same* as that in Short Division.

Solve the following examples by Long Division:

2. Divide 4672 by 8. *Ans.* 584.
3. Divide 97636 by 7. *Ans.* 13948.
4. Divide 37863 by 9. *Ans.* 4207.
5. Divide 394064 by 11. *Ans.* 35824.
6. Divide 38889 by 9. *Ans.* 4321.
7. Divide 67536 by 12. *Ans.* 5628.

8. How many times is 23 contained in 17158?

OPERATION.

23)17158(746
161
105
92
138
138

SOLUTION. — As 23 is not contained in the first *two* figures of the dividend, we find how many times it is contained in 171, as the first partial dividend. 23 is contained in 171, 7 times, which we place in the quotient on the right of the dividend. Then we multiply the divisor 23, by the quotient figure 7, and subtract the product 161, from the part of the dividend used, and we have a remainder of 10. To this remainder we annex the next figure of the dividend, making 105 for the second partial dividend. 23 is contained in 105, 4 times, which we place in the quotient. Multiplying and subtracting as before, we have a remainder of 13, to which we annex the next figure of the dividend, making 138 for the third partial dividend. 23 is contained in 138, 6 times; multiplying and subtracting as before, nothing remains, and we have for the entire quotient, 746.

From the preceding illustrations we derive the following general rule:

Rule. I. *Write the divisor at the left of the dividend, as in short division.*

II. *Divide the least number of the left hand figures in the dividend that will contain the divisor one or more times, and place the quotient at the right of the dividend, with a line between them.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend.*

IV. *Divide as before, until all the figures of the dividend have been brought down and divided.*

V. *If any partial dividend will not contain the divisor, place a cipher in the quotient, annex*

the next figure of the dividend, and divide as before.

VI. *When there is a remainder after dividing all the figures of the dividend, it must be written in the quotient, with the divisor underneath.*

1. If any remainder is *equal to*, or *greater* than, the divisor, the quotient figure is too *small*, and must be increased.
2. If the product of the divisor by the quotient figure is *greater* than the partial dividend, the quotient figure is too *large*, and must be diminished.

Proof. *The same as in short division.*

71. The operations in long division consist of five principal steps, viz.:

1. Writing down the numbers.
2. Finding how many times one is contained in the other.
3. Multiplying.
4. Subtracting.
5. Annexing to the remainder another figure from the dividend.

9. Find how many times 18 is contained in 36838.

OPERATION.		PROOF.	
Dividend. Quotient.			
Divisor. 18)36838(2046$\frac{10}{18}$		2046	Quotient.
36		18	Divisor.
83		16368	
72		2046	
118		36828	
108		10	Remainder.
10	Remainder.	36838	Dividend.

10. Divide 79638 by 36.

OPERATION.

$$
\begin{array}{r}
36)79638(2212\tfrac{6}{36} \\
72 \\
\hline
76 \\
72 \\
\hline
43 \\
36 \\
\hline
78 \\
72 \\
\hline
6 \text{ Rem.}
\end{array}
$$

Divide 93975 by 84.

OPERATION.

$$
\begin{array}{r}
84)93975(1118\tfrac{63}{84} \\
84 \\
\hline
99 \\
84 \\
\hline
157 \\
84 \\
\hline
735 \\
672 \\
\hline
63 \text{ Rem.}
\end{array}
$$

11. Divide 408722 by 136.

OPERATION.

$$
\begin{array}{r}
136)408722(3005 \\
408 \\
\hline
722 \\
680 \\
\hline
42 \text{ Rem.}
\end{array}
$$

Divide 104762 by 109.

OPERATION.

$$
\begin{array}{r}
109)104762(961 \\
981 \\
\hline
666 \\
654 \\
\hline
122 \\
109 \\
\hline
13 \text{ Rem.}
\end{array}
$$

12. Divide 178464 by 16. *Ans.* 11154.

13. Divide 15341 by 29. *Ans.* 529.

14. Divide 463554 by 39. *Ans.* 11886.

15. Divide 1299123 by 17. *Ans.* 76419.

16. Divide 161700 by 15. *Ans.* 10780.

17. Divide 47653 by 24.

18. Divide 765431 by 42.

		Quotients.	Rem.
19.	Divide 6783 by 15.	452	3.
20.	Divide 7831 by 18.	435	1.
21.	Divide 9767 by 22.	443	21.
22.	Divide 7654 by 24.	318	22.
23.	Divide 767500 by 23.	33369	13.
24.	Divide 250765 by 34.	7375	15.
25.	Divide 5571489 by 43.	129569	22.
26.	Divide 153598 by 29.	5296	14.
27.	Divide 301147 by 63.	4780	7.
28.	Divide 40231 by 75.	536	31.
29.	Divide 52761878 by 126.	418745	8.
30.	Divide 92550 by 25.	3702	
31.	Divide 7461300 by 95.	78540	
32.	Divide 1193288 by 45.	26517	23.
33.	Divide 5973467 by 243.	24582	41.
34.	Divide 69372168 by 342.	202842	204.
35.	Divide 863256 by 736.	1172	664.
36.	Divide 1893312 by 912.	2076	
37.	Divide 833382 by 207.	4026	
38.	Divide 52847241 by 607.	87063	
39.	Divide 13699840 by 342.	40058	4.
40.	Divide 946656 by 1038.	912	
41.	Divide 46447786 by 1234.	37640	26.
42.	Divide 28101418481 by 1107.	25385201	974.
43.	Divide 48288058 by 3094.	15607	
44.	Divide 47254149 by 4674.	10110	9.

45. A man bought 114 acres of land for $4104. What was the average price per acre? *Ans.* $36.

46. Nine thousand dollars was paid to 75 operatives. What did each receive? *Ans.* $120.

47. There are 24 hours in a day. How many days in 11424 hours? *Ans.* 476 days.

48. In one hogshead there are 63 gallons. How many hogsheads in 6615 gallons? *Ans.* 105 hogsheads.

49. If a man travels 48 miles a day, how long will it take him to travel 1296 miles? *Ans.* 27 days.

50. If a person can count 8677 in an hour, how long will it take him to count 38369694? *Ans.* 4422 hours.

51. If it costs $5987520 to construct a railroad 576 miles long, what will be the average cost per mile?

Ans. $10395.

52. A certain railroad is 287 miles in length, and cost $5572470. What was the average cost per mile?

Ans. 19416\frac{78}{287}$

53. A garrison consumed 1712 barrels of flour in 107 days. How much was that per day? *Ans.* 16 barrels.

54. How long would it take a vessel to sail from New York to China, supposing the distance to be 9072 miles, and the ship to average 144 miles a day? *Ans.* 63 days.

55. How long could 27 men subsist on a stock of provisions that would last 1 man 3456 days? *Ans.* 128 days.

56. A drover received $10362 for 314 head of cattle. What was their average value per head? *Ans.* $33.

57. If 42864 pounds of cotton are packed in 94 bales, what is the average weight of each bale?

Ans. 456 pounds.

58. If a field containing 42 acres produces 1659 bushels of wheat, what will be the number of bushels produced per acre? *Ans.* 39$\frac{1}{2}$ bushels.

59. In what time will a reservoir containing 109440 gallons be emptied by a pump discharging 608 gallons per hour? *Ans.* 180 hours.

CONTRACTIONS.

EXAMPLES.

72. When the divisor is 10, 100, 1000, etc.

1. Divide 374 by 10.

OPERATION.

1|0)37|4

Quotient, 37 . . . 4 Rem.

or, $37\frac{4}{10}$, *Ans.*

SOLUTION. — We have shown that to remove a figure one place toward the left by annexing a cipher increases its value tenfold, or multiplies it by 10. So, on the contrary, by cutting off or taking away the right hand figure of a number, each of the other figures is removed one place toward the right, and, consequently, the value of each is diminished tenfold, or divided by 10.

For similar reasons, if we cut off *two* figures, we divide by 100, if *three*, we divide by 1000, and so on.

Hence the rule:

Rule. *From the right hand of the dividend cut off as many figures as there are ciphers in the divisor. Under the figures so cut off place the divisor, and the whole will form the quotient.*

	Quotients.	Rem.
2. Divide 13705 by 100.	137	5.
3. Divide 50670 by 100.	506	70.
4. Divide 320762 by 1000.	320	762.
5. Divide 14030731 by 10000.	1403	731.
6. Divide 9021300640 by 100000.	90213	640.

7. A man sold 100 acres of land for $3725. What did he receive an acre? *Ans.* $37\frac{25}{100}$.

8. Bought 1000 barrels of flour for $6080. What did it cost me a barrel? *Ans.* $6\frac{80}{1000}$.

9. Paid $12500 for 10000 bushels of wheat. What was the cost per bushel? *Ans.* $1\frac{2500}{10000}$.

73. When there are ciphers on the right hand of the divisor.

1. Divide 437661 by 800.

OPERATION.

8|00)4376|61

547 . . . 61 Rem.

SOLUTION. — We resolve 800 into the factors 8 and 100, and divide first by 100, by cutting off the two right hand figures of the dividend, and we have a quotient of 4376, with a remainder of 61. We next divide by 8, and obtain 547 for a quotient; and the entire quotient is $547\frac{61}{800}$.

2. Divide 34716 by 900.

OPERATION.

9|00)347|16

Quotient, 38 . . . 516 Rem.

or, $38\frac{516}{900}$, *Ans.*

SOLUTION. — Dividing as in the last example, first by 100 and then by 9, we have a quotient of 38, and a remainder of 5, which we prefix to the figures cut off from the dividend, making a true remainder of 516, and the entire quotient $38\frac{516}{900}$.

Rule. I. *Cut off the ciphers from the right of the divisor, and the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor, and the result will be the quotient. If there is a remainder after this division, prefix it to the figures cut off from the dividend, and this will form the true remainder.*

	Quotients.	Rem.
3. Divide 46820 by 400.	117	20.
4. Divide 130627 by 800.	163	227.
5. Divide 76173 by 320.	238	13.
6. Divide 378000 by 1200.	315	
7. Divide 674321 by 11200.	60	2321.
8. Divide 64613214 by 4000.	16153	1214.
9. Divide 146200 by 430.	340	

EXAMPLES IN THE PRECEDING RULES.

1. A speculator bought at different times 320 acres, 175 acres, 87 acres, and 32 acres of land, and afterward sold 467 acres. How many acres had he left?

Ans. 147 acres.

2. Two men travel in opposite directions; one travels 31 miles a day, the other 43 miles a day. How far apart will they be in 12 days? *Ans.* 888 miles.

3. A tobacconist has 6324 pounds of tobacco, which he wishes to pack in boxes containing 62 pounds each. How many boxes must he procure to obtain it?

Ans. 102 boxes.

4. A farmer sold 15 tons of hay at $9 a ton, and 25 cords of wood at $4 a cord, and wished to divide the amount equally among 5 creditors. What would each receive? *Ans.* $47.

5. If you deposit 216 cents each week in a savings-bank, and take out 89 cents a week, how much money will you have in bank at the end of 36 weeks? *Ans.* $45.72.

6. The product of two numbers is 8928, and one of the numbers is 72. What is the other number?

Ans. 124.

7. The dividend is 7280, and the quotient is 208. What is the divisor? *Ans.* 35.

8. What is the remainder after dividing 876437 by 16900? *Ans.* 14537.

9. A man sold 6 horses at $125 each, 25 head of cattle at $30 each, and with the proceeds bought land at $25 an acre. How many acres did he buy?

Ans. 60 acres.

10. If a mechanic receives $784 a year for labor, and his expenses are $426 a year, how much can he save in 6 years? *Ans.* $2148.

11. A farmer sold 40 bushels of wheat at $2 a bushel, and 16 cords of wood at $3 a cord. He received 15 yards of cloth at $4 a yard, and the remainder in money. How much money did he receive? *Ans.* $68.

12. How many pounds of cheese, worth 10 cents a pound, can be bought for 22 pounds of butter, worth 15 cents a pound? *Ans.* 33 pounds.

13. If 56 yards of cloth cost $336, how much will 12 yards cost at the same rate? *Ans.* $72.

14. If 100 barrels of flour cost $600, what will 350 barrels cost at the same rate? *Ans.* $2100.

15. How long can 60 men subsist on an amount of food that will last 1 man 7620 days? *Ans.* 127 days.

16. If I buy 225 barrels of flour for $1125, and sell the same for $1800, how much do I gain on each barrel? *Ans.* $3.

17. A man sold his house and lot for $5670, and took his pay in bank stock at $90 a share. How many shares did he receive? *Ans.* 63 shares.

18. How many pounds of tea, worth 75 cents a pound, ought a man to receive in exchange for 27 bushels of oats, worth 50 cents a bushel? *Ans.* 18 pounds.

19. The quotient of one number divided by another is 40, the divisor is 364, and the remainder 120. What is the dividend? *Ans.* 14680.

20. How many tons of hay, at $12 a ton, must be given for 21 cows at $24 apiece? *Ans.* 42 tons.

21. Bought 150 barrels of flour for $1050, and sold 107 barrels of it at $9 a barrel, and the remainder at $7 a barrel. Did I gain or lose, and how much? *Ans.* Gained $214.

22. A mechanic earns $45 a month, and his necessary expenses are $27 a month. How long will it take him to pay for a farm of 50 acres, at $27 an acre? *Ans.* 75 months.

CANCELLATION.

74. Cancellation is the process of rejecting equal factors from numbers sustaining to each other the relation of dividend and divisor.

Whenever the dividend and divisor are each composite numbers, the factors common to both may first be rejected without altering the final result.

EXAMPLES.

1. What is the quotient of 48 divided by 24?

OPERATION.

$$\frac{48}{24} = \frac{\cancel{3} \times \cancel{8} \times 2}{\cancel{3} \times \cancel{8}} = 2, \ \textit{Ans.}$$

SOLUTION. — We first *indicate* the operation to be performed by writing the dividend above a line, and the divisor below it. We resolve 48 into the factors 3, 8, and 2, and 24 into the factors 3 and 8. We next cancel the factors 3 and 8, which are common to the dividend and divisor, and we have left the factor 2, in the dividend, which is the quotient.

1. When all the factors or numbers in the dividend are canceled, 1 must be retained.
2. Rejecting a factor from any number is dividing the number by that factor.
3. When a factor is canceled, the unit, 1, is supposed to take its place.
4. *One* factor in the dividend will cancel only *one equal* factor in the divisor.
5. If all the factors or numbers of the divisor are canceled, the product of the remaining factors of the dividend will be the quotient.

2. Divide the product of $12 \times 8 \times 6$ by $8 \times 4 \times 3$.

$$\frac{\overset{3}{\cancel{12}} \times \cancel{8} \times \overset{2}{\cancel{6}}}{\cancel{8} \times \cancel{4} \times \cancel{3}} = \frac{3 \times 2}{1} = 6, \ \textit{Ans.}$$

3. Divide the product of $25 \times 18 \times 4 \times 3$ by $7 \times 6 \times 5 \times 3$.

$$\frac{\overset{5}{\cancel{25}} \times \overset{3}{\cancel{18}} \times 4 \times \cancel{3}}{7 \times \cancel{6} \times \cancel{5} \times \cancel{3}} = \frac{5 \times 3 \times 4}{7} = \frac{60}{7} = 8\tfrac{4}{7}, \ \textit{Ans.}$$

4. Divide the product of $36 \times 10 \times 7$ by $14 \times 5 \times 9$.

Ans. 4.

5. What is the quotient of $21 \times 8 \times 40 \times 3$ divided by $12 \times 7 \times 20$? *Ans.* 12.

6. What is the quotient of $64 \times 18 \times 9$ divided by $30 \times 27 \times 4$? *Ans.* $3\frac{1}{5}$.

7. Divide the product of $120 \times 44 \times 6$ by $60 \times 11 \times 8$.

8. How many days' work, at 84 cents a day, will pay for 36 bushels of corn worth 56 cents a bushel?

9. A farmer exchanged 45 bushels of potatoes, worth 30 cents a bushel, for 15 pounds of tea. What was the tea worth a pound? *Ans.* 90 cents.

10. A grocer bought 120 pounds of cheese, at 9 cents a pound, and paid for it in molasses, at 45 cents a gallon. How many gallons of molasses paid for the cheese?

11. Gave 12 barrels of flour, at \$7 a barrel, for hay worth \$18 a ton. How many tons of hay was the flour worth? *Ans.* $4\frac{2}{3}$ tons.

12. Sold 8 firkins of butter, each weighing 56 pounds, at 15 cents a pound, and received in payment 3 boxes of tea, each containing 40 pounds. What was the tea worth a pound? *Ans.* 56 cents.

13. A man took 6 loads of apples to market, each load containing 14 barrels, and each barrel 3 bushels. He sold them at 50 cents a bushel, and received in payment 9 barrels of sugar, each weighing 210 pounds. What was the sugar worth a pound? *Ans.* $6\frac{2}{3}$ cents.

14. A grocer sold 12 boxes of soap, each containing 51 pounds, at 10 cents a pound; he received in payment a certain number of barrels of potatoes, each containing 3 bushels, at 30 cents a bushel. How many barrels did he receive? *Ans.* 68 barrels.

15. A man sold 4 loads of barley, each load containing 60 bushels, at 70 cents a bushel, and received in payment 2 pieces of cloth, each piece containing 35 yards. What was the cloth worth a yard? *Ans.* \$2.40.

PROBLEMS IN SIMPLE INTEGRAL NUMBERS.

75. The four operations that have now been considered, viz. Addition, Subtraction, Multiplication, and Division, are all the operations that can be performed upon numbers, and hence they are called the *Fundamental Rules*.

76. In all cases, the numbers operated upon and the results obtained sustain to each other the relation of a whole to its parts.

1. *In Addition*, the numbers added are the parts, and the sum or amount is the whole.

Thus, in $4 + 5 = 9$, 4 and 5 are the parts and 9 is the whole.

2. *In Subtraction*, the subtrahend and remainder are the parts, and the minuend is the whole.

Thus, in $9 - 4 = 5$, 4 and 5 are the parts and 9 the whole.

3. *In Multiplication*, the multiplicand denotes the value of one part, the multiplier the number of parts, and the product the total value of the whole number of parts.

Thus, in $5 \times 2 = 10$, 2 is the value of one part, 5 denotes the number of parts, and 10 the total value of the parts.

4. *In Division*, the dividend denotes the total value of the whole number of parts, the divisor the value of one part, and the quotient the number of parts; or the divisor the number of parts, and the quotient the value of one part.

Thus, in $10 \div 5 = 2$, 10 denotes the total value of all the parts, 5 the value of one part, and 2 the number of parts; or 2 the value of one part, and 5 the number of parts.

FRACTIONS.

DEFINITIONS, NOTATION, AND NUMERATION.

77. When a unit is divided into 2 equal parts, one of the parts is called *one half*.

When a unit is divided into 3 equal parts, one of the parts is called *one third*, and two of the parts are called *two thirds*.

When a unit is divided into 4 equal parts, one of the parts is called *one fourth*, two of the parts are called *two fourths*, and three of the parts are called *three fourths*.

When a unit is divided into 5 equal parts, one of the parts is called *one fifth*, two of the parts are called *two fifths*, three of the parts, *three fifths*, etc.

Since *one half*, *one third*, *one fourth*, and all other *equal parts* of an *integer* or *whole thing*, are each in themselves *entire* and complete, the *parts of a unit* thus used are called *fractional units;* and the numbers formed from them, *fractional numbers*.

78. A **Fractional Unit** is one of the equal parts of an integral unit.

79. A **Fraction** is a fractional unit, or a collection of fractional units.

80. Fractional units take their name, and their value, from the *number* of parts into which the integral unit is divided.

Thus, if we divide an orange into 2 equal parts, the parts are called *halves;* if into 3 equal parts, *thirds;* if into 4 equal parts, *fourths,* etc.; and each *third* is less in value than each *half,* and each *fourth* less than each *third;* and the greater the *number* of parts, the less their *value.*

The parts of a fraction are expressed by figures; thus,

One half	is written	$\frac{1}{2}$	One fifth	is written	$\frac{1}{5}$
One third	"	$\frac{1}{3}$	Two fifths	"	$\frac{2}{5}$
Two thirds	"	$\frac{2}{3}$	One seventh	"	$\frac{1}{7}$
One fourth	"	$\frac{1}{4}$	Three eighths	"	$\frac{3}{8}$
Two fourths	"	$\frac{2}{4}$	Five ninths	"	$\frac{5}{9}$
Three fourths	"	$\frac{3}{4}$	Eight tenths	"	$\frac{8}{10}$

To write a fraction, therefore, two integers are required, one written above the other with a line between them.

81. The **Denominator** of a fraction is the number below the line. It shows into how many parts the integer or unit is divided, and determines the *value* of the *fractional unit.*

82. The **Numerator** is the number above the line. It numbers the *fractional units,* and shows *how many* are taken.

Thus, if one dollar is divided into 4 equal parts, the parts are called *fourths,* the fractional unit being *one fourth,* and three of these parts are called *three fourths* of a dollar, and may be written

$\frac{3}{4}$ the number of parts or *fractional units* taken.
4 the number of parts or *fractional units* into which the dollar is divided.

83. The **Terms** of a fraction are the numerator and denominator, taken together.

84. Fractions indicate *division*, the numerator corresponding to the dividend, and the denominator to the divisor.

85. The **Value** of a fraction is the quotient of the numerator divided by the denominator.

Thus, the quotient of 4 divided by 5 is $\frac{4}{5}$, or $\frac{4}{5}$ expresses the quotient of which $\left\{\begin{array}{l}4 \text{ is the dividend.}\\5 \text{ is the divisor.}\end{array}\right.$

EXAMPLES.

86. To find any fractional part of a number.

1. What is $\frac{1}{2}$ of 8?

OPERATION.

$2)\overline{8}$
$\quad 4$

SOLUTION. — It is the quotient of 8 divided by 2, which is 4; or, it is a number which, taken 2 times, will make 8, which is 4.

OPERATION.

$3)\overline{9}$
$\quad 3$
$\quad 2$
$\quad\overline{6}$

2. What is $\frac{2}{3}$ of 9?

SOLUTION. — One third of 9 is 3, 2 thirds of 9 is 2 times 3, or 6.

Rule. *To obtain one half, one third, one fourth, or any fractional part of a number, divide that number by the denominator of the fraction expressing the parts; and to obtain any given number of such parts, multiply that part by the number of parts expressed by the numerator of the same fraction.*

3. What is $\frac{1}{4}$ of 12? $\frac{3}{4}$ of 12? $\frac{1}{8}$ of 12? $\frac{1}{6}$ of 12?
4. What is $\frac{1}{5}$ of 20? $\frac{2}{5}$ of 20? $\frac{3}{5}$ of 20? $\frac{4}{5}$ of 20?

5. What is $\frac{1}{8}$ of 40? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

6. What is $\frac{2}{7}$ of 21? $\frac{5}{7}$ of 35? $\frac{6}{7}$ of 49?

7. What is $\frac{1}{9}$ of 63? $\frac{2}{9}$ of 27? $\frac{4}{9}$ of 36? $\frac{5}{9}$ of 45? $\frac{7}{9}$ of 81? $\frac{8}{9}$ of 63?

8. What is $\frac{1}{12}$ of 48? $\frac{5}{12}$? $\frac{7}{12}$?

9. If a pound of coffee costs 15 cents, what will $\frac{1}{3}$ of a pound cost? $\frac{2}{3}$ of a pound?

10. A farmer having 60 sheep, sold $\frac{1}{5}$ of them to one man, and $\frac{3}{5}$ to another? How many did he sell to both?

11. A boy having 48 cents, spent $\frac{3}{8}$ of them. How many had he left?

12. Paid 108 dollars for a horse, and $\frac{9}{12}$ as much for a carriage. What did the carriage cost?

87. To find what fractional part one number is of another number.

1. What part of 5 is 3?

$1 = \frac{1}{5}$ of 5.

$3 = 3 \times \frac{1}{5}$ of $5 = \frac{3}{5}$ of 5.

SOLUTION. — 1 is 1 fifth of 5, 3 is 3 times 1 fifth of 5, or 3 fifths of 5.

Rule. *Make the number of which the part is to be found the numerator, and the part the denominator.*

2. What part of 6 is 3? 4? 5? 1?

3. What part of 9 is 2? 3? 5? 6? 1? 4?

4. What part of 10 is 7? 6? 3? 1? 9? 8? 4?

5. What part of 12 is 3? 5? 6? 8? 9? 7? 10? 11?

6. What part of 14 is 5? 7? 9? 3? 6? 11? 8? 15?

7. What part of 15 bushels are 3 bushels? 7 bushels? 9 bushels? 11 bushels?

8. What part of $18 is $7? $5? $9? $17?

9. If 6 oranges cost 30 cents, what part of 30 cents will 1 orange cost? 2 oranges? 3 oranges? 5 oranges?

EXAMPLES IN WRITING AND READING FRACTIONS.

Express the following fractions by figures:

1. Nine *twelfths.* *Ans.* $\frac{9}{12}$.
2. Eleven *fifteenths.* *Ans.* $\frac{11}{15}$.
3. Twenty-four *forty-ninths.* *Ans.* $\frac{24}{49}$.
4. Forty-four *sixty-ninths.* *Ans.* $\frac{44}{69}$.
5. One hundred twenty *four hundred fiftieths.*

Read the following fractions:

6. $\frac{7}{15}$, $\frac{12}{40}$, $\frac{27}{66}$, $\frac{84}{120}$, $\frac{142}{847}$, $\frac{15}{869}$, $\frac{156}{90}$, $\frac{450}{817}$.
7. $\frac{8}{5}$, $\frac{2}{25}$, $\frac{19}{86}$, $\frac{19}{360}$, $\frac{190}{860}$, $\frac{1}{925}$, $\frac{125}{80}$, $\frac{728}{8}$, $\frac{654}{54}$.
8. If the fractional unit is 28, express 9 fractional units; 16; 17; 22; 27.
9. If the fractional unit is 96, express 27 fractional units; 42; 75.

88. Fractions are distinguished as **Proper** and **Improper.**

A *Proper Fraction* is one whose numerator is less than its denominator; its value is less than the unit, 1.

Thus, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{9}{10}$, $\frac{72}{84}$ are proper fractions.

An *Improper Fraction* is one whose numerator equals or exceeds its denominator; its value is never less than the unit, 1.

Thus, $\frac{7}{7}$, $\frac{8}{5}$, $\frac{15}{4}$, $\frac{35}{8}$, $\frac{50}{16}$, $\frac{180}{90}$ are improper fractions.

89. A **Mixed Number** is a number expressed by an integer and a fraction.

Thus, $4\frac{1}{4}$, $17\frac{13}{25}$, $9\frac{3}{10}$ are mixed numbers.

REDUCTION.

90. The **Reduction** of a fraction is the process of changing its form without altering its value.

A fraction is in its *lowest terms* when no number greater than 1 will exactly divide both numerator and denominator without a remainder.

EXAMPLES.

91. To reduce fractions to their lowest terms.

1. Reduce $\frac{2}{4}$ to its lowest terms.

OPERATION.

$$\frac{2 \div 2}{4 \div 2} = \frac{1}{2}.$$

SOLUTION. — It is plain that the numerator 2, and the denominator 4, are both divisible by 2, without remainders. The terms thus obtained, viz. 1, the numerator, and 2, the denominator, are not both divisible by any number greater than 1, and therefore are the *smallest* terms by which the value of $\frac{2}{4}$ can be expressed.

Reduce to their lowest terms:

2. $\frac{3}{6}$. 4. $\frac{6}{8}$. 6. $\frac{2}{4}$. 8. $\frac{6}{12}$. 10. $\frac{9}{18}$.
3. $\frac{5}{10}$. 5. $\frac{8}{16}$. 7. $\frac{7}{14}$. 9. $\frac{5}{20}$. 11. $\frac{10}{30}$.

12. Reduce $\frac{48}{60}$ to its lowest terms.

OPERATION.

$$\frac{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 5} = \frac{4}{5}$$

SOLUTION. — Separate both terms of the fraction into their factors, and cancel as many of the factors as possible (**74**) before the division is made.

Rule. *Cancel the common factors of both terms of the fraction. The fraction is then expressed in its lowest terms.*

Reduce to their lowest terms:

13. $\frac{60}{72}$. *Ans.* $\frac{5}{6}$. 17. $\frac{144}{156}$. *Ans.* $\frac{12}{13}$. 21. $\frac{825}{980}$. *Ans.* $\frac{55}{64}$.
14. $\frac{72}{120}$. *Ans.* $\frac{3}{5}$. 18. $\frac{240}{322}$. *Ans.* $\frac{120}{161}$. 22. $\frac{82}{256}$. *Ans.* $\frac{1}{8}$.
15. $\frac{98}{112}$. *Ans.* $\frac{7}{8}$. 19. $\frac{216}{288}$. *Ans.* $\frac{3}{4}$. 23. $\frac{171}{180}$. *Ans.* $\frac{19}{20}$.
16. $\frac{126}{162}$. *Ans.* $\frac{7}{9}$. 20. $\frac{455}{315}$. *Ans.* $\frac{1}{7}$. 24. $\frac{5600}{7555}$. *Ans.* $\frac{1120}{1511}$.

92. To change an improper fraction to a whole or mixed number.

1. In $\frac{12}{4}$ how many times 1?

OPERATION.
$\frac{12}{4} = 12 \div 4 = 3$

SOLUTION. — Performing the division indicated, the result is 3; $\frac{12}{4}$ equals 3.

2. How many times 1 in $\frac{15}{3}$? In $\frac{18}{6}$? In $\frac{20}{5}$?
3. How many times 1 in $\frac{28}{7}$? In $\frac{24}{6}$? In $\frac{32}{8}$?
4. How many times 1 in $\frac{64}{8}$? In $\frac{60}{10}$? In $\frac{48}{6}$?
5. How many times 1 in $\frac{72}{9}$? In $\frac{66}{11}$? In $\frac{96}{12}$?

When the denominator is not an exact divisor of the numerator the result will be a mixed number.

6. In $\frac{16}{7}$ how many times 1?

OPERATION.
$7)\overline{16}$
$2\frac{2}{7}$, *Ans.*

SOLUTION. — Performing the division indicated, the result is $2\frac{2}{7}$; $\frac{16}{7}$ equals $2\frac{2}{7}$.

Rule. *Divide the numerator by the denominator.*

7. In $\frac{196}{8}$ how many times 1? *Ans.* $24\frac{1}{2}$.
8. In $\frac{228}{12}$ of a year how many years? *Ans.* 19.
9. In $\frac{1284}{12}$ of a pound how many pounds? *Ans.* 107.
10. In $\frac{672}{112}$ of a mile how many miles? *Ans.* 6.
11. In $\frac{7007}{88}$ of a rod how many rods? *Ans.* $212\frac{1}{8}$.
12. In $\frac{2245}{10}$ of a dollar how many dollars?
13. Reduce $\frac{96}{16}$ to a whole number. *Ans.* 6.
14. Reduce $\frac{140}{25}$ to a mixed number. *Ans.* $5\frac{3}{5}$.
15. Reduce $\frac{324}{18}$ to a whole number. *Ans.* 18.
16. Reduce $\frac{1456}{24}$ to a mixed number. *Ans.* $60\frac{2}{3}$.
17. Change $\frac{8246}{48}$ to a mixed number. *Ans.* $67\frac{5}{8}$.
18. Change $\frac{2184}{42}$ to a whole number. *Ans.* 52.
19. Change $\frac{676}{52}$ to a whole number. *Ans.* 13.
20. Change $\frac{1380}{68}$ to a mixed number.
21. Change $\frac{3850}{51}$ to a mixed number.

93. To reduce a whole or a mixed number to an improper fraction.

1. How many thirds in 4?

OPERATION.

$1 = \frac{3}{3}$

$\frac{3}{3} \times 4 = \frac{12}{3}$

SOLUTION. — In 1 there are 3 *thirds*, in 4 there are 4 times 3 *thirds*, or 12 *thirds*.

2. How many *fourths* in 2? In 3? In 5?
3. How many *halves* in 5? In 7? In 8? In 9?
4. How many *sixths* in 3? In 5? In 7? In 10?
5. How many *tenths* in 4? In 8? In 9? In 6?
6. How many *fifths* in 2 whole oranges? In 4? In 5?
7. How many *eighths* in 4 whole dollars? In 5? In 6?
8. In $3\frac{5}{8}$ dollars how many *eighths* of a dollar?

OPERATION.

$3\frac{5}{8}$

$\frac{8}{24} + 5 = \frac{29}{8}$

SOLUTION. — Since in 1 dollar there are 8 *eighths*, in 3 dollars there are 3 times 8 *eighths*, or 24 *eighths*, and 5 *eighths* added, make $\frac{29}{8}$.

Rule. *Multiply the whole number by the denominator of the fraction (in case of a mixed number, add the product to the numerator); and under the result write the denominator.*

9. Reduce $6\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{27}{4}$.
10. Reduce $7\frac{5}{9}$ to an improper fraction. *Ans.* $\frac{68}{9}$.
11. Reduce 15 to a fraction whose denominator is 7. *Ans.* $\frac{105}{7}$.
12. Reduce 120 to *twelfths*. *Ans.* $\frac{1440}{12}$.
13. In $242\frac{3}{8}$ of an acre how many *thirds* of an acre?
14. In $757\frac{7}{8}$ bushels how many *eighths*? *Ans.* $\frac{6057}{8}$.
15. In 24 pounds how many *sixteenths*? *Ans.* $\frac{384}{16}$.
16. In 52 weeks how many *sevenths*? *Ans.* $\frac{364}{7}$.
17. Change $14\frac{9}{24}$ to an improper fraction. *Ans.* $\frac{345}{24}$.

94. To reduce two or more fractions to a common denominator.

A **Common Denominator** is a denominator common to two or more fractions.

1. Reduce $\frac{1}{4}$ and $\frac{2}{3}$ to fractions having a common denominator.

OPERATION.

$\frac{1}{4} = \frac{3}{12}$

$\frac{1}{3} = \frac{4}{12}$ $\therefore$ $\frac{2}{3} = \frac{8}{12}$

SOLUTION. — 12 is exactly divisible by 4 and 3, and may therefore be taken for a common denominator. Since in 1 there are $\frac{12}{12}$, in $\frac{1}{4}$ of 1 there must be $\frac{1}{4}$ of $\frac{12}{12}$, or $\frac{3}{12}$; and in $\frac{2}{3}$ of 1 there must be $\frac{2}{3}$ of $\frac{12}{12}$, or $\frac{8}{12}$; $\frac{1}{4}$ and $\frac{2}{3}$ are equal to $\frac{3}{12}$ and $\frac{8}{12}$.

Reduce to common denominators:

2. $\frac{3}{4}$ and $\frac{1}{6}$.
3. $\frac{1}{3}$ and $\frac{3}{5}$.
4. $\frac{1}{4}$ and $\frac{5}{8}$.
5. $\frac{2}{5}$ and $\frac{1}{4}$.

6. Reduce $\frac{5}{6}$ and $\frac{3}{5}$ to a common denominator.

OPERATION.

$5 \times 5 = 25$

$6 \times 5 = \overline{30}$

$3 \times 6 = 18$

$5 \times 6 = \overline{30}$

SOLUTION. — We multiply the terms of the first fraction by the denominator of the second, and the terms of the second fraction by the denominator of the first. This reduces each fraction to the same denominator, 30, for each new denominator will be the product of the given denominators.

Rule. *Multiply both terms of each fraction by the denominators of all the other fractions.*

Mixed numbers must first be reduced to improper fractions.

Reduce to common denominators:

7. $\frac{4}{7}$ and $\frac{3}{4}$. *Ans.* $\frac{16}{28}$, $\frac{21}{28}$.
8. $\frac{4}{5}$ and $\frac{7}{8}$. *Ans.* $\frac{32}{40}$, $\frac{35}{40}$.
9. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. *Ans.* $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$.
10. $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{4}{7}$. *Ans.* $\frac{35}{105}$, $\frac{63}{105}$, $\frac{60}{105}$.
11. $1\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$. *Ans.* $\frac{108}{72}$, $\frac{54}{72}$, $\frac{56}{72}$.
12. $\frac{7}{10}$, $2\frac{1}{4}$, and $\frac{5}{6}$. *Ans.* $\frac{168}{240}$, $\frac{540}{240}$, $\frac{200}{240}$.

ADDITION.

95. The denominator of a fraction determines the value of the fractional unit; hence,

1. If two or more fractions have the same denominator, their numerators express fractional units of the same value.

2. If two or more fractions have different denominators, their numerators express fractional units of different values.

And since only units of the same value can be united into one sum, it follows that —

3. Fractions can be added only when they have the same fractional unit or common denominator.

EXAMPLES.

To add fractions.

1. What is the sum of $\frac{1}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{2}{5}$?

OPERATION.

$$\frac{1}{5}+\frac{3}{5}+\frac{4}{5}+\frac{2}{5}=\frac{1+3+4+2}{5}=\frac{10}{5}=2$$

SOLUTION. — When fractions have a common denominator, their sum is found by adding their numerators, and placing the sum over the common denominator. $1 + 3 + 4 + 2 = 10$, the sum of the numerators; placing this sum over the common denominator 5, we have $\frac{10}{5} = 2$, the required sum.

2. What is the sum of $\frac{3}{10}$, $\frac{4}{10}$, and $\frac{7}{10}$?

3. What is the sum of $\frac{2}{7}$, $\frac{5}{7}$, $\frac{1}{7}$, and $\frac{6}{7}$?

4. What is the sum of $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{8}$, $\frac{5}{8}$, and $\frac{2}{8}$?

5. A boy paid $\frac{5}{6}$ of a dollar for a pair of gloves, $\frac{2}{6}$ of a dollar for a knife, and $\frac{1}{6}$ of a dollar for a slate. What did he pay for all?

6. A father distributed money among his children, as follows: to the first he gave $\frac{5}{12}$ of a dollar, to the second $\frac{3}{12}$, and to the third $\frac{7}{12}$. What did he give to all?

7. What is the sum of $\frac{3}{4}$ and $\frac{2}{9}$?

OPERATION.

$\frac{3}{4} + \frac{2}{9} = \frac{27}{36} + \frac{8}{36} = \frac{35}{36}$, *Ans.*

SOLUTION. — As the given fractions have not a common denominator, we reduce them to the same fractional unit (**94**), and then add their numerators, $27 + 8 = 35$; placing the sum over the common denominator, 36, we obtain $\frac{35}{36}$.

Rule. I. *When the given fractions have the same denominator, add the numerators, and under the sum write the common denominator.*

II. *When they have not the same denominator, reduce them to a common denominator, and then add as before.*

If the amount is an improper fraction, reduce it to a whole or a mixed number.

Add:

8. $\frac{2}{3}$, $\frac{4}{5}$. *Ans.* $1\frac{7}{15}$.
9. $\frac{1}{6}$, $\frac{5}{7}$. *Ans.* $\frac{37}{42}$.
10. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{8}$. *Ans.* $1\frac{7}{8}$.
11. $\frac{3}{7}$, $\frac{1}{2}$, $\frac{2}{9}$. *Ans.* $1\frac{109}{126}$.
12. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{7}$. *Ans.* $1\frac{121}{140}$.
13. $\frac{5}{6}$, $\frac{5}{7}$, $\frac{5}{8}$. *Ans.* $2\frac{209}{168}$.

To add mixed numbers.

14. What is the sum of $14\frac{2}{5}$, $21\frac{1}{2}$, and $9\frac{3}{4}$?

OPERATION.

$14\frac{2}{5} = 14\frac{8}{20}$
$21\frac{1}{2} = 21\frac{10}{20}$
$9\frac{3}{4} = 9\frac{15}{20}$
$45\frac{33}{20}$, *Ans.*

SOLUTION. — By reducing the fractions to a common denominator, and adding them, we obtain $\frac{33}{20}$, or $1\frac{13}{20}$, which, added to the sum of the integral numbers, equals $45\frac{13}{20}$.

Rule. *Add the fractions and integers separately, and then add their sums.*

If the mixed numbers are small, they may be reduced to improper fractions, and then added in the usual way.

15. What is the sum of $3\frac{1}{4}$, $12\frac{5}{6}$, and $25\frac{2}{3}$? *Ans.* $41\frac{3}{4}$.

16. Bought three pieces of cloth containing $45\frac{1}{2}$, $38\frac{1}{3}$, and 35 yards. How many yards were there in the 3 pieces?

SUBTRACTION.

96. The process of subtracting one fraction from another is based upon the following principles:

1. One number can be subtracted from another only when the two numbers have the same unit value. Hence,

2. In subtraction of fractions, the minuend and subtrahend must have a common denominator.

EXAMPLES.

To subtract one fraction from another.

1. From $\frac{9}{12}$ subtract $\frac{5}{12}$.

OPERATION. $\frac{9}{12} - \frac{5}{12} = \frac{9-5}{12} = \frac{4}{12}$

SOLUTION. — Since the fractions have a common denominator, the difference is obtained by subtracting the less numerator 5 from the greater 9, and writing the result over the common denominator 12; we thus obtain $\frac{4}{12}$, the required difference.

2. From $\frac{7}{8}$ subtract $\frac{3}{8}$.

3. From $\frac{11}{15}$ subtract $\frac{9}{15}$.

4. Subtract $\frac{17}{24}$ from $\frac{21}{24}$.

5. James had $\frac{7}{8}$ of a bushel of walnuts, and sold $\frac{4}{8}$ of them. How many had he left?

6. Harvey had $\frac{9}{10}$ of a dollar, and spent $\frac{5}{10}$ of a dollar. What part had he left?

7. Subtract $\frac{3}{7}$ from $\frac{2}{3}$.

OPERATION. $\frac{2}{3} - \frac{3}{7} = \frac{14}{21} - \frac{9}{21} = \frac{5}{21}$, *Ans.*

SOLUTION. — As the given fractions have not a common denominator, we first reduce them to the same fractional unit (**94**), and then subtract the less numerator 9 from the greater 14, and write the result over the common denominator 21. We thus obtain $\frac{5}{21}$, the required difference.

Rule. I. *When the fractions have the same denominator, subtract the less numerator from the greater, and place the result over the common denominator.*

II. *When they have not a common denominator, reduce them to a common denominator before subtracting.*

8. From $\frac{7}{8}$ take $\frac{3}{4}$. *Ans.* $\frac{1}{8}$.
9. From $\frac{2}{3}$ take $\frac{4}{9}$. *Ans.* $\frac{2}{9}$.
10. From $\frac{5}{7}$ take $\frac{2}{5}$. *Ans.* $\frac{11}{35}$.
11. From $\frac{9}{10}$ take $\frac{1}{6}$. *Ans.* $\frac{11}{15}$.
12. Subtract $\frac{5}{9}$ from $\frac{3}{4}$. *Ans.* $\frac{7}{36}$.
13. Subtract $\frac{9}{20}$ from $\frac{4}{5}$. *Ans.* $\frac{7}{20}$.
14. Subtract $\frac{4}{5}$ from $1\frac{1}{3}$. *Ans.* $\frac{3}{65}$.
15. Subtract $\frac{7}{18}$ from $1\frac{1}{2}$. *Ans.* $1\frac{2}{36}$.
16. Subtract $1\frac{1}{24}$ from $\frac{2}{3}$. *Ans.* $\frac{5}{24}$.
17. Subtract $\frac{21}{80}$ from $\frac{7}{8}$. *Ans.* $\frac{7}{40}$.

To subtract mixed numbers.

18. From $9\frac{1}{3}$ take $2\frac{3}{4}$.

OPERATION.

$9\frac{1}{3} = 9\frac{4}{12}$
$2\frac{3}{4} = 2\frac{9}{12}$
$6\frac{7}{12}$, *Ans.*

SOLUTION. — We first reduce the fractional parts, $\frac{1}{3}$ and $\frac{3}{4}$, to a common denominator 12. Since we cannot take $\frac{9}{12}$ from $\frac{4}{12}$, we add $1 = \frac{12}{12}$ to $\frac{4}{12}$, which makes $\frac{16}{12}$. $\frac{9}{12}$ taken from $\frac{16}{12}$ leaves $\frac{7}{12}$. We now add 1 to the 2 in the subtrahend; 3 taken from 9 leaves 6. We thus obtain $6\frac{7}{12}$, the difference required.

Rule. *Reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately.*

19. From $24\frac{3}{4}$ take $17\frac{1}{2}$. *Ans.* $7\frac{1}{4}$.
20. From $147\frac{2}{8}$ take $49\frac{1}{4}$. *Ans.* $98\frac{5}{12}$.
21. From $75\frac{1}{3}$ take $40\frac{3}{5}$. *Ans.* $34\frac{11}{15}$.
22. From $63\frac{3}{10}$ take $22\frac{3}{6}$. *Ans.* $40\frac{4}{5}$.
23. Bought flour at $\$6\frac{1}{2}$ a barrel, and sold it at $\$7\frac{3}{5}$ a barrel. What was the gain per barrel?

Ans. $\frac{9}{10}$ of a dollar.

MULTIPLICATION.

EXAMPLES.

97. To multiply a fraction by an integer.

1. If one pound of sugar costs $\frac{1}{8}$ of a dollar, what will 3 pounds cost?

OPERATION. $3 \times \frac{1}{8} = \frac{3}{8}$ $\frac{3}{8}$ of a dollar.

SOLUTION. — Three pounds will cost 3 times $\frac{1}{8}$, or $\frac{3}{8}$ of a dollar.

2. If 1 horse eats $\frac{2}{9}$ of a ton of hay in one month, how much will 4 horses eat in the same time?

3. At $\frac{3}{5}$ of a dollar a bushel, what will be the cost of 2 bushels of pears? Of 3 bushels? Of 5 bushels?

4. How many are 3 times $\frac{2}{5}$? 5 times $\frac{3}{4}$? 4 times $\frac{1}{6}$? 6 times $\frac{2}{7}$? 9 times $\frac{1}{10}$? 8 times $\frac{3}{5}$?

5. If one yard of cloth costs $\frac{5}{6}$ of a dollar, what will 3 yards cost?

FIRST OPERATION. $\frac{5}{6} \times 3 = \frac{15}{6} = 2\frac{1}{2}$

SECOND OPERATION. $\frac{5}{6} \times 3 = \frac{5}{2} = 2\frac{1}{2}$

SOLUTION. — In the first operation we multiply the fraction by 3, by multiplying its numerator by 3, obtaining $\frac{15}{6} = 2\frac{1}{2}$. In this case the *value* of the fractional unit remains the same, but we multiply the *number taken*, 3 times. In the second operation we multiply the fraction by 3, by dividing its denominator by 3, obtaining $\frac{5}{2} = 2\frac{1}{2}$. In this case the *value* of the fractional unit is multiplied 3 times, but the *number taken* is the same. Hence,

Rule. *To multiply a fraction by an integer, multiply its numerator, or divide its denominator, by that number.*

Always divide the denominator when it is exactly divisible by the multiplier.

6. Multiply $\frac{6}{7}$ by 5. *Ans.* $4\frac{2}{7}$.
7. Multiply $\frac{7}{9}$ by 4. *Ans.* $3\frac{1}{9}$.
8. Multiply $\frac{9}{10}$ by 6. *Ans.* $5\frac{2}{5}$.
9. Multiply $\frac{12}{27}$ by 9. *Ans.* 4.
10. Multiply $\frac{9}{24}$ by 3. *Ans.* $1\frac{1}{8}$.
11. Multiply $\frac{30}{42}$ by 14. *Ans.* 10.

To multiply a mixed number by an integer.

12. Multiply $4\frac{1}{3}$ by 5.

OPERATION.

$4\frac{1}{3}$
5 Or,
$1\frac{2}{3}$ $4\frac{1}{3} = \frac{13}{3}$
20 $\frac{13}{3} \times 5 = \frac{65}{3} = 21\frac{2}{3}$
$21\frac{2}{3}$

SOLUTION. — To multiply a mixed number, we first multiply the fractional part, then the integer, and then add the two products; $5 \times \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$; and $5 \times 4 = 20$, which added to $1\frac{2}{3}$, gives $21\frac{2}{3}$, the required result. Or, we may reduce the mixed number to an improper fraction, and then multiply it.

Rule. I. *Multiply the fractional parts and the integer separately, and add the results. Or,*

II. *Reduce the mixed number to an improper fraction, and proceed as in* **97.**

13. Multiply $6\frac{3}{4}$ by 8. *Ans.* 54.

14. Multiply $9\frac{4}{5}$ by 7. *Ans.* $68\frac{3}{5}$.

15. If a man earns $\$1\frac{7}{8}$ in 1 day, how much will he earn in 10 days? *Ans.* $\$18\frac{3}{4}$.

16. What will 14 yards of cloth cost, at $\frac{5}{7}$ of a dollar a yard? *Ans.* \$10.

17. At $\$3\frac{1}{4}$ a cord, what will be the cost of 20 cords of wood? *Ans.* \$65.

18. If a man earns $\$3\frac{3}{4}$ a day, how much will he earn in 7 days?

98. To multiply an integer by a fraction.

1. At \$9 a barrel, what will $\frac{2}{3}$ of a barrel of flour cost?

OPERATION.

$\$9 \div 3 = \$3$
$2 \times \$3 = \$6$, *Ans.*

SOLUTION. — Since 1 barrel of flour costs \$9, $\frac{1}{3}$ of a barrel will cost $\frac{1}{3}$ of \$9, or \$3, and $\frac{2}{3}$ of a barrel will cost $2 \times \$3 = \$6$.

Rule. *Divide the integer by the denominator, and multiply the quotient by the numerator of the fraction.*

2. If a yard of cloth is worth \$8, what is $\frac{3}{4}$ of a yard worth?

3. If an acre of land produces 25 bushels of wheat, how much will $\frac{1}{5}$ of an acre produce? $\frac{3}{5}$ of an acre? $\frac{4}{5}$ of an acre?

4. If a man earns \$20 in a month, what can he earn in $\frac{1}{2}$ of a month? In $\frac{3}{5}$ of a month? In $\frac{7}{10}$ of a month? In $\frac{3}{4}$ of a month?

5. If a ton of hay costs \$12, what will $\frac{1}{4}$ of a ton cost? $\frac{3}{4}$ of a ton? $\frac{2}{3}$ of a ton? $\frac{5}{6}$ of a ton?

6. At \$60 an acre, what will $\frac{1}{5}$ of an acre of land cost?

7. Multiply 45 by $\frac{3}{4}$. *Ans.* $33\frac{3}{4}$.

8. Multiply 68 by $\frac{4}{5}$. *Ans.* $54\frac{2}{5}$.

9. Multiply 105 by $\frac{7}{15}$. *Ans.* 49.

10. Multiply 480 by $\frac{5}{8}$. *Ans.* 300.

11. At \$16 a ton, what will be the cost of $\frac{3}{4}$ of a ton of hay? *Ans.* \$12.

12. If a village lot is worth \$340, what is $\frac{3}{4}$ of it worth? *Ans.* \$255.

13. If a hogshead of sugar is worth \$75, what is $\frac{11}{12}$ of it worth? *Ans.* $\$68\frac{3}{4}$.

14. If an acre of land produces 114 bushels of oats, how many bushels will $\frac{9}{16}$ of an acre produce?

Ans. $64\frac{1}{8}$ bushels.

15. If a man travels 47 miles in a day, how far can he travel in $\frac{5}{8}$ of a day? *Ans.* $29\frac{3}{8}$ miles.

99. To multiply a fraction by a fraction.

1. If a bushel of apples is worth $\frac{1}{4}$ of a dollar, what is $\frac{1}{2}$ of a bushel worth?

OPERATION.

$\frac{1}{4} = \frac{2}{8}$

$\frac{1}{2}$ of $\frac{2}{8} = \frac{1}{8}$

SOLUTION. — Since 1 bushel is worth $\frac{1}{4}$ of a dollar, $\frac{1}{2}$ of a bushel is worth $\frac{1}{2}$ of $\frac{1}{4}$ of a dollar; $\frac{1}{4}$ equals $\frac{2}{8}$, and $\frac{1}{2}$ of $\frac{2}{8}$ is $\frac{1}{8}$; $\frac{1}{8}$ of a bushel is worth $\frac{1}{8}$ of a dollar.

2. If a yard of cloth costs $\frac{1}{2}$ a dollar, what will $\frac{1}{3}$ of a yard cost?

3. When oats are worth $\frac{1}{8}$ of a dollar a bushel, what is $\frac{1}{4}$ of a bushel worth?

4. If a man owns $\frac{1}{2}$ of a vessel, and he sells $\frac{1}{3}$ of his share, what part of the vessel does he sell?

5. At $\frac{2}{3}$ of a dollar a bushel, what will $\frac{3}{4}$ of a bushel of corn cost?

OPERATION.

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$, *Ans.*

SOLUTION. — Since 1 bushel costs $\frac{2}{3}$ of a dollar, $\frac{3}{4}$ of a bushel will cost $\frac{3}{4}$ times $\frac{2}{3}$ of a dollar. By multiplying the numerator 2 by the numerator 3, we obtain the numerator 6 of the product; and by multiplying the denominator 3 by the denominator 4, we obtain the denominator 12 of the product, and thus we have $\frac{6}{12} = \frac{1}{2}$ for the required product.

Rule. *Find the product of the numerators for a new numerator, and the product of the denominators for a new denominator, and reduce the result to its lowest terms.*

6. Multiply $\frac{1}{2}$ by $\frac{3}{5}$. *Ans.* $\frac{3}{10}$.
7. Multiply $\frac{3}{4}$ by $\frac{3}{7}$. *Ans.* $\frac{9}{28}$.
8. Multiply $\frac{4}{5}$ by $\frac{6}{7}$. *Ans.* $\frac{24}{35}$.
9. Multiply $\frac{3}{8}$ by $\frac{5}{6}$. *Ans.* $\frac{5}{24}$.
10. Multiply $\frac{7}{12}$ by $\frac{5}{7}$. *Ans.* $\frac{5}{12}$.
11. What is the product of $\frac{2}{5}$, $\frac{1}{8}$, and $\frac{3}{4}$? *Ans.* $\frac{1}{10}$.
12. What is the product of $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{2}{5}$? *Ans.* $\frac{5}{72}$.
13. What is the product of $\frac{7}{9}$, $\frac{2}{7}$, and $\frac{5}{10}$? *Ans.* $\frac{1}{9}$.
14. What is the product of $\frac{13}{24}$ and $\frac{16}{26}$? *Ans.* $\frac{1}{3}$.

15. What is the product of $\frac{5}{6}$, $1\frac{1}{2}$, and $5\frac{4}{5}$?

OPERATION.

$\frac{5}{6} \times 1\frac{1}{2} \times 5 \times \frac{4}{5} =$

$\frac{5}{6} \times \frac{3}{2} \times \frac{5}{1} \times \frac{4}{5} = \frac{10}{2} = 5$, *Ans.*

SOLUTION. — When integers or mixed numbers occur among the given factors, they may be reduced to improper fractions before multiplying. An integer may be reduced to the form of a

fraction by writing 1 for its denominator. We therefore change $1\frac{1}{2}$ to $\frac{3}{2}$, and 5 to $\frac{5}{1}$, and proceed as before.

16. What is the product of $\frac{3}{7}$, $\frac{2}{5}$, and $2\frac{3}{8}$? *Ans.* $\frac{1}{8}\frac{8}{8}$.

17. What is the product of 3, $\frac{9}{10}$, and $\frac{5}{8}$? *Ans.* $2\frac{1}{4}$.

18. What is the product of $\frac{3}{7}$, $\frac{5}{11}$, and $\frac{2}{2}\frac{2}{5}$? *Ans.* $\frac{6}{35}$.

19. Find the value of $\frac{2}{3}$ of $\frac{5}{8}$ multiplied by $\frac{3}{5}$ of $\frac{9}{11}$.

OPERATION.

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{8}_{4}} \times \frac{\cancel{3}}{\cancel{5}} \times \frac{9}{11} = \frac{9}{44}, \textit{Ans.}$$

SOLUTION. — When the same factors occur in both numerator and denominator of fractions to be multiplied, they may be canceled (**74**) and the remaining factors only used; thus 5 and 3 being found in both the numerators and denominators may be omitted in multiplying, and 2, being a factor of 8, may be canceled, leaving 9 in the numerator and $4 \times 11 = 44$ in the denominator. Hence the answer is $\frac{9}{44}$.

Fractions with the word *of* between them are sometimes called *compound fractions*. The word *of* is simply an equivalent for the sign multiplication, (×).

20. Multiply $\frac{3}{4}$ of $\frac{6}{7}$ by $\frac{4}{9}$ of $\frac{1}{6}$. *Ans.* $\frac{1}{21}$.

21. Multiply $\frac{4}{5}$ of 3 by $\frac{7}{8}$ of $2\frac{1}{2}$. *Ans.* $5\frac{1}{4}$.

22. What is the product of $\frac{9}{10}$, $\frac{1}{2}$ of $\frac{5}{9}$, and $1\frac{1}{3}$? *Ans.* $\frac{1}{3}$.

23. What is the product of $\frac{6}{7}$ of $\frac{7}{11}$ by $5\frac{1}{2}$. *Ans.* 3.

24. What is the value of $\frac{3}{5}$ times $\frac{1}{6}$ of $\frac{2}{3}$ of 10? *Ans.* $\frac{2}{3}$.

25. What is the value of $\frac{5}{12}$ of $\frac{4}{5}$ times $\frac{1}{4}$ of $3\frac{3}{7}$? *Ans.* $\frac{2}{7}$.

26. At $\frac{3}{8}$ of a dollar a bushel, what will $\frac{2}{3}$ of a bushel of corn cost? *Ans.* $\frac{1}{4}$ of a dollar.

27. When peaches are worth $\frac{9}{10}$ of a dollar a bushel, what is $\frac{5}{9}$ of a bushel worth? *Ans.* $\frac{1}{2}$ dollar.

28. Jane having $\frac{4}{5}$ of a yard of silk gave $\frac{3}{4}$ of it to her sister. What part a yard did she give her sister?

Ans. $\frac{3}{5}$ of a yard.

29. When pears are worth $\frac{7}{8}$ of a dollar a basket, what is $\frac{4}{7}$ of $\frac{3}{4}$ of a basket worth? *Ans.* $\frac{3}{8}$ of a dollar.

30. A man owning $\frac{1}{2}\frac{3}{0}$ of a ship, sold $\frac{5}{8}$ of his share. What part of the whole ship did he sell? *Ans.* $\frac{1}{8}\frac{5}{2}$.

31. A grocer having $\frac{1}{2}\frac{5}{1}$ of a hogshead of molasses sold $\frac{7}{10}$ of it. What part of a hogshead remained?

DIVISION.

EXAMPLES.

100. To divide a fraction by an integer.

1. If 3 pounds of cherries cost $\frac{1}{2}$ of a dollar, what will 1 pound cost?

OPERATION.
$\frac{1}{2} \div 3 = \frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

SOLUTION. — If 3 pounds cost $\frac{1}{2}$ of a dollar, 1 pound will cost $\frac{1}{3}$ of $\frac{1}{2}$, or $\frac{1}{6}$ of a dollar.

Rule. *Divide the numerator or multiply the denominator by the integer.*

We divide the numerator when it is exactly divisible by the divisor; otherwise we multiply the denominator.

2. If 2 pounds of coffee cost $\frac{1}{8}$ of a dollar, what will 1 pound cost?

3. If 5 marbles cost $\frac{1}{8}$ of a dollar, what will 1 marble cost?

4. If $\frac{1}{5}$ of a barrel of flour is equally divided among 6 persons, what part of a barrel will each receive?

5. If $\frac{1}{7}$ of a box of tea is equally distributed among 8 persons, what part of a box will each receive?

6. Paid $\frac{8}{9}$ of a dollar for 4 pounds of butter. What was the cost per pound?

7. Divide $\frac{6}{7}$ by 3. *Ans.* $\frac{2}{7}$.
8. Divide $\frac{4}{9}$ by 4. *Ans.* $\frac{1}{9}$.
9. Divide $\frac{10}{13}$ by 5. *Ans.* $\frac{2}{13}$.
10. Divide $\frac{5}{16}$ by 5. *Ans.* $\frac{1}{16}$.
11. Divide $\frac{3}{4}$ of $\frac{8}{9}$ by 12. *Ans.* $\frac{1}{18}$.
12. Divide $\frac{4}{5}$ of $\frac{3}{8}$ by 6. *Ans.* $\frac{1}{15}$.
13. Divide $4\frac{1}{5}$ by 7.

OPERATION.
$4\frac{1}{5} = \frac{21}{5}$
$\frac{21}{5} \div 7 = \frac{3}{5}$, *Ans.*

SOLUTION. — We reduce the mixed number to an improper fraction, and then divide as before.

14. Divide $3\frac{4}{5}$ by 4. *Ans.* $\frac{19}{20}$.

15. Divide $6\frac{3}{8}$ by 9. *Ans.* $\frac{17}{24}$.

16. Divide $\frac{1}{2}$ of $2\frac{1}{4}$ by 3. *Ans.* $\frac{3}{8}$.

17. Divide $8\frac{7}{10}$ by 12. *Ans.* $\frac{29}{40}$.

18. Divide $13\frac{3}{4}$ by 10. *Ans.* $1\frac{3}{8}$.

19. Divide $\frac{5}{6}$ of 8 by 20. *Ans.* $\frac{1}{3}$.

20. If 6 men agree to share equally $\frac{3}{4}$ of a bushel of grapes, what part of a bushel will each man have?

Ans. $\frac{1}{8}$.

21. If 5 yards of sheeting cost $\frac{9}{10}$ of a dollar, what will 1 yard cost? *Ans.* $\frac{9}{50}$ of a dollar.

22. If 8 bushels of apples cost $\$5\frac{3}{5}$, what will 1 bushel cost? *Ans.* $\frac{2}{3}$ of a dollar.

23. If $\frac{1}{2}$ of 10 pounds of butter cost $\$1\frac{1}{4}$, what will 1 pound cost? *Ans.* $\frac{1}{4}$ of a dollar.

24. A man distributed $\frac{18}{20}$ of a dollar equally among 6 children. What part of a dollar did he give to each?

25. If $\frac{2}{3}$ of 9 cords of wood cost $\$12\frac{3}{8}$, what will 1 cord cost?

101. To divide an integer by a fraction.

1. At $\frac{2}{3}$ of a dollar a yard, how many yards of ribbon can be bought for 2 dollars?

OPERATION.

$2 \div \frac{2}{3} = 2 \times \frac{3}{2}$

$= \frac{6}{2} = 3$

SOLUTION. — As many yards can be bought as $\frac{2}{3}$ of a dollar is contained times in 2 dollars. 3 yards of ribbon can be bought for 2 dollars.

Rule. *Multiply the integer by the denominator, and divide it by the numerator of the fraction.*

2. When potatoes are $\frac{3}{4}$ of a dollar a bushel, how many bushels can be bought for 2 dollars? For 4 dollars? For 6 dollars?

3. At $\frac{4}{5}$ of a dollar a bushel, how many bushels of corn can be bought for 16 dollars?

4. If a man spends $\frac{1}{8}$ of a dollar a day for cigars, how long will it take him to spend 3 dollars? 5 dollars? 6 dollars?

5. Divide 18 by $\frac{2}{3}$. *Ans.* 27.

6. Divide 14 by $\frac{2}{7}$. *Ans.* 49.

7. Divide 11 by $\frac{5}{9}$. *Ans.* $19\frac{4}{5}$.

8. Divide 75 by $\frac{9}{10}$. *Ans.* $83\frac{1}{3}$.

9. Divide 120 by $\frac{6}{11}$. *Ans.* 220.

10. Divide 96 by $\frac{12}{17}$. *Ans.* 136.

11. Divide 226 by $\frac{9}{25}$. *Ans.* $627\frac{7}{9}$.

12. Divide 28 by $4\frac{2}{3}$.

OPERATION.

$4\frac{2}{3} = \frac{14}{3}$

$28 \times \frac{3}{14} = 6$, *Ans.*

SOLUTION. — We reduce the mixed number to an improper fraction, $\frac{14}{3}$, and then divide the integer in the same manner as by a proper fraction.

13. Divide 16 by $2\frac{1}{4}$. *Ans.* $7\frac{1}{9}$.

14. Divide 42 by $3\frac{1}{2}$. *Ans.* 12.

15. Divide 112 by $6\frac{2}{5}$. *Ans.* $17\frac{1}{2}$.

16. Divide 180 by $7\frac{1}{8}$. *Ans.* $25\frac{5}{19}$.

17. Divide 425 by $\frac{5}{7}$. *Ans.* 595.

18. Divide 318 by $\frac{6}{23}$. *Ans.* 1219.

19. When potatoes are $\frac{7}{8}$ of a dollar a bushel, how many bushels can be bought for $10? *Ans.* $12\frac{8}{7}$ bu.

20. Divide 9 bushels of corn among a number of families, giving them $\frac{3}{16}$ of a bushel each. How many families will receive a share? *Ans.* 48.

21. At $2\frac{4}{5}$ a cord, how many cords of wood can be bought for 27 dollars? *Ans.* $9\frac{9}{14}$ cords.

22. If a horse eats $\frac{5}{9}$ of a bushel of oats in a day, in how many days will he eat 20 bushels? *Ans.* 36 days.

23. If a man walks $2\frac{9}{10}$ miles an hour, how many hours will he require to walk 48 miles? *Ans.* $16\frac{8}{29}$ hours.

24. At $\frac{1}{16}$ of a dollar a pound, how many pounds of rice can be bought for 3 dollars? *Ans.* 48 pounds.

102. To divide a fraction by a fraction.

1. At $\frac{2}{5}$ of a dollar a pound, how many pounds of tea can be bought for $\frac{4}{5}$ of a dollar?

OPERATION.

$\frac{4}{5} \div \frac{2}{5} = 4 \div 2 = 2$

SOLUTION.— Two *fifths* are contained in 4 *fifths* 2 times. Two pounds can be bought for $\frac{4}{5}$ of a dollar.

When fractions have a common denominator, division may be performed by dividing the numerator of the dividend by the numerator of the divisor.

2. How many pineapples, at $\frac{3}{10}$ of a dollar each, can be bought for $\frac{6}{10}$ of a dollar? For $\frac{9}{10}$? For $\frac{12}{10}$?

3. If a horse eats $\frac{2}{7}$ of a bushel of oats in 1 day, in how many days will it eat $\frac{4}{7}$ of a bushel? $\frac{6}{7}$? $\frac{10}{7}$? $\frac{14}{7}$?

4. At $\frac{1}{8}$ of a dollar a bushel, how many bushels of apples can be bought for $\frac{2}{8}$ of a dollar? For $\frac{4}{8}$? For $\frac{6}{8}$?

5. At $\frac{2}{5}$ of a dollar a pound, how many pounds of tea can be bought for $\frac{3}{4}$ of a dollar?

OPERATION.

$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$, *Ans.*

SOLUTION. — As many pounds can be bought as $\frac{4}{5}$ of a dollar, the price of 1 pound, is contained times in $\frac{3}{4}$ of a dollar. By inverting the terms of the divisor the two fractions will stand in such relation to each other, that we can find the product of the two upper numbers for the numerator of the quotient, and of the two lower numbers for the denominator.

Rule. I. *Reduce integers and mixed numbers to improper fractions.*

II. *Invert the terms of the divisor, and proceed as in multiplication.*

6. Divide $\frac{9}{10}$ by $\frac{3}{10}$. *Ans.* 3.
7. Divide $\frac{1}{2}$ by $\frac{1}{4}$. *Ans.* 2.
8. Divide $\frac{5}{8}$ by $\frac{2}{3}$. *Ans.* $\frac{15}{16}$.
9. Divide $\frac{7}{8}$ by $\frac{2}{5}$. *Ans.* $2\frac{3}{16}$.
10. How many times is $\frac{7}{19}$ contained in $\frac{17}{19}$? *Ans.* $2\frac{3}{7}$.
11. How many times is $\frac{6}{7}$ contained in $\frac{3}{5}$? *Ans.* $\frac{7}{10}$.

12. How many times is $\frac{1}{2}$ contained in $\frac{5}{16}$? *Ans.* $1\frac{7}{8}$.
13. Divide $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{7}{8}$. *Ans.* $\frac{3}{7}$.
14. Divide $\frac{4}{5}$ of $\frac{5}{8}$ by $\frac{7}{20}$. *Ans.* $1\frac{3}{7}$.
15. Divide $\frac{11}{15}$ by $\frac{1}{7}$ of $\frac{2}{3}$. *Ans.* $7\frac{7}{10}$.
16. Divide $\frac{3}{4}$ of $\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{7}{8}$. *Ans.* $1\frac{1}{14}$.
17. At $\frac{1}{7}$ of a dollar a pound, how many pounds of sugar can be bought for $\frac{4}{5}$ of a dollar? *Ans.* $5\frac{3}{5}$ pounds.
18. At $\frac{7}{10}$ of a dollar a gallon, how much oil can be bought for $\frac{1}{2}$ of a dollar? *Ans.* $\frac{5}{7}$ of a gallon.
19. At $\frac{4}{5}$ of $\frac{1}{8}$ of a dollar a yard, how many yards of ribbon can be bought for $\frac{7}{10}$ of a dollar? *Ans.* $2\frac{5}{8}$ yards.
20. At $\frac{7}{8}$ of a dollar a yard, how many yards of silk can be bought for $\frac{5}{8}$ of a dollar? *Ans.* $2\frac{1}{7}$ yards.
21. A man owning $\frac{5}{8}$ of a copper mine, divided his share equally among his sons, giving them $\frac{5}{16}$ each. How many sons had he? *Ans.* 2.
22. If $\frac{9}{7}$ of a bushel of pears costs $\frac{5}{8}$ of a dollar, what will 1 bushel cost? *Ans.* $\frac{7}{10}$ of a dollar.
23. How much corn, at $\frac{5}{8}$ of a dollar a bushel, can be bought for $\frac{3}{8}$ of a dollar? *Ans.* $\frac{3}{4}$ of a bushel.

PROMISCUOUS EXAMPLES.

1. In $25\frac{9}{16}$ pounds how many 16ths of a pound?
2. Reduce $\frac{421}{86}$ to a mixed number. *Ans.* $11\frac{25}{86}$.
3. Reduce $\frac{1568}{2088}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
4. In $\frac{1852}{8}$ of a day how many days?
5. Change 42 pounds to *sevenths* of a pound.
6. Reduce $21\frac{4}{5}$ to an improper fraction. *Ans.* $\frac{109}{5}$.
7. Reduce $126\frac{3}{8}$ to *thirds*. *Ans.* $\frac{3}{8}\frac{8}{3}\frac{0}{}$.
8. Reduce $\frac{288}{860}$ to its lowest terms. *Ans.* $\frac{4}{5}$.
9. Reduce $\frac{1}{2}$ and $\frac{4}{5}$ to a common denominator.
10. Reduce 36 to a fraction whose denominator is 12.
11. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$? *Ans.* $1\frac{7}{8}$.
12. Add $\frac{9}{10}$, $\frac{1}{2}$, and $3\frac{1}{5}$. *Ans.* $4\frac{3}{5}$.

13. Reduce $\frac{9}{16}$, $\frac{7}{8}$, and $\frac{3}{4}$ to a common denominator.

14. Sold $9\frac{3}{8}$ cords of wood to one man, and $12\frac{9}{16}$ to another. How much did I sell to both?

15. Paid $\$87\frac{9}{10}$ for a horse, and $\$62\frac{1}{2}$ for a wagon. How much more was paid for the horse than for the wagon? *Ans.* $\$25\frac{3}{5}$.

16. A farmer having $234\frac{15}{16}$ acres of land, sells at one time $42\frac{3}{4}$ acres, at another time $61\frac{3}{8}$, and at another $70\frac{1}{2}$ acres. How many acres has he left? *Ans.* $60\frac{5}{16}$ acres.

17. A speculator bought 120 bushels of wheat for $\$136\frac{3}{5}$, and sold it for $\$197\frac{5}{8}$. What did he gain?

18. Bought 12 pounds of coffee at $\frac{1}{8}$ of a dollar a pound, and 9 pounds of tea at $\frac{4}{5}$ of a dollar a pound. What was the cost of the whole? *Ans.* $\$8\frac{7}{10}$.

19. Bought 10 bushels of wheat, at $\$1\frac{1}{3}$ a bushel, and 14 bushels of corn, at $\frac{5}{7}$ of a dollar a bushel. Which cost the more, and how much? *Ans.* The wheat, $\$3\frac{1}{3}$.

20. Paid $\$12$ for some cloth, at the rate of $\frac{3}{4}$ of a dollar a yard. How many yards were purchased?

21. If 8 oranges cost $\frac{2}{3}$ of $\$1\frac{1}{5}$, what will 1 orange cost? *Ans.* $\frac{1}{10}$ of a dollar.

22. A man bought $\frac{7}{9}$ of a farm and sold $\frac{3}{8}$ of his share. What part of the whole farm did he sell? What part had he left? *Ans.* Sold $\frac{14}{27}$.

23. If the dividend is $\frac{20}{21}$, and the quotient $\frac{5}{7}$, what is the divisor? *Ans.* $1\frac{1}{3}$.

24. If the divisor is $\frac{9}{16}$, and the quotient $3\frac{1}{3}$, what is the dividend? *Ans.* $2\frac{4}{13}$.

25. How many barrels of apples can be bought for $\$18$, at $\$1\frac{3}{16}$ a barrel? *Ans.* $15\frac{3}{19}$ barrels.

26. If the smaller of two fractions is $\frac{4}{11}$, and the difference $\frac{2}{5}$, what is the greater? *Ans.* $\frac{42}{55}$.

27. If the sum of two fractions is $1\frac{1}{3}$, and one of them is $\frac{9}{20}$, what is the other? *Ans.* $\frac{27}{40}$.

DECIMAL FRACTIONS.

NOTATION AND NUMERATION.

103. Decimal Fractions are fractions which have for their denominator 10, 100, 1000, or 1 with any number of ciphers annexed, and are usually written like the orders of integers.

Decimal fractions are commonly called *decimals*.

Since $\frac{1}{10} = \frac{10}{100}$, $\frac{1}{100} = \frac{10}{1000}$, etc., the denominators of decimal fractions increase and decrease in a tenfold ratio, in the same manner as simple numbers.

104. In the formation of decimals a unit is divided into 10 equal parts, called *tenths;* each of these *tenths* is divided into 10 other equal parts, called *hundredths;* each of these *hundredths* into 10 other equal parts, called *thousandths*, and so on.

Since the denominators of decimal fractions increase and decrease by the scale of 10, in the same way as simple numbers, in writing decimals the denominators may be omitted.

105. The Decimal point (.) is always placed before decimal figures to distinguish them from integers.

Thus	$\frac{6}{10}$	is expressed	.6
	$\frac{54}{100}$	" "	.54
	$\frac{279}{1000}$	" "	.279

.5 is 5 tenths, which = $\frac{1}{10}$ of 5 units;
.05 is 5 hundredths, " = $\frac{1}{10}$ of 5 tenths;
.005 is 5 thousandths, " = $\frac{1}{10}$ of 5 hundredths.

And universally, the value of a figure in any decimal place is $\frac{1}{10}$ the value of the same figure in the next left-hand place.

106. The relation of decimals and integers to each other is clearly shown by the following table:

DECIMAL NUMERATION TABLE.

5 Millions.	7 Hund. of thousands.	3 Tens of thousands.	2 Thousands.	7 Hundreds.	5 Tens.	4 Units.	. *Decimal Point.*	5 Tenths.	7 Hundredths.	3 Thousandths.	2 Ten-thousandths.	5 Hundred-thousandths.	6 Millionths.

By examining this table we see that —

Tenths are expressed by one figure.
Hundredths are expressed by two figures.
Thousandths are expressed by three figures.

107. Since the denominator of tenths is 10, of hundredths 100, of thousandths 1000, and so on, a decimal may be expressed by writing the numerator only; but in this case the numerator or decimal must always contain as many decimal places as are equal to the number of ciphers in the denominator; and the denominator of a decimal will always be the unit 1, with as many ciphers annexed as are equal to the number of figures in the decimal or numerator.

The decimal point must never be omitted.

EXAMPLES.

1. Express in decimal form seven tenths. *Ans.* .7.
2. Write twenty-five hundredths. *Ans.* .25.
3. Write nine hundredths. *Ans.* .09.
4. Write one hundred twenty-five thousandths.
5. Write eighteen thousandths.
6. Write fifty-eight hundredths.
7. Write two hundred thirty-six thousandths.

Read the following decimals:

8.	.06	.143	.0005	.479.
9.	.34	.037	.3240	.00341.
10.	.80	.472	.1026	.102367.

108. A mixed decimal number is a number consisting of integers and decimals.

Thus, 71.406 consists of the integral part, 71, and the decimal part, .406; it is read the same as $71\frac{406}{1000}$, 71 and 406 thousandths.

1. Write twenty-four, and four tenths. *Ans.* 24.4.
2. Write thirty-two, and five hundredths.
3. Write seventy-six, and forty-six thousandths.
4. Write one hundred twelve, and one hundred ninety thousandths. *Ans.* 112.190.
5. Write sixty-three, and forty-four ten-thousandths.
6. Write five, and 5 hundred-thousandths.
7. Write sixteen, and 21 ten-thousandths.
8. Write eight, and 234 hundred-thousandths.

Read the following numbers:

9.	42.08	50.002	640.00010.
10.	81.110	161.0301	7.4230.
11.	120.0342	14.42000	3.01206.

109. From the foregoing explanations and illustrations we derive the following important principles:

Principles of Decimal Notation and Numeration.

1. The *value* of any decimal figure depends upon its *place* from the decimal point.

Thus .3 is ten times .03.

2. *Prefixing* a cipher to a decimal decreases its value the same as dividing it by ten.

Thus .03 is $\frac{1}{10}$ the value of .3.

3. *Annexing* a cipher to a decimal does not alter its value, since it does not change the *place* of the significant figures of the decimal.

Thus, $\frac{6}{10}$, or .6, is the same as $\frac{60}{100}$, or .60.

4. Decimals increase from right to left, and decrease from left to right, in a tenfold ratio; and therefore they may be added, subtracted, multiplied, and divided in the same manner as whole numbers.

5. The denominator of a decimal, though never expressed, is always the unit 1, with as many ciphers annexed as there are figures in the decimal.

6. To read decimals requires two numerations; first, *from* units, to find the name of the denominator, and second, *towards* units, to find the value of the numerator.

110. From the principles upon which the writing and reading of decimals depend, we derive the following rules:

Rule for Decimal Notation. I. *Write the decimal in the same way as a whole number, placing ciphers where necessary to give each significant figure its true local value.*

II. *Place the decimal point before the first figure.*

Rule for Decimal Numeration. I. *Numerate from the decimal point, to determine the denominator.*

II. *Numerate towards the decimal point, to determine the numerator.*

III. *Read the decimal as a whole number, giving it the name of its lowest decimal unit, or right hand figure.*

EXAMPLES.

1. Write 325 ten-thousandths. *Ans.* .0325.
2. Write 400 ten-thousandths.
3. Write 117 ten-thousandths.
4. Write 10 ten-thousandths. *Ans.* .0010.
5. Write 250 millionths. *Ans.* .000250.
6. Write 1200 ten-thousandths.
7. Write 9 hundred-thousandths. *Ans.* .00009.

Read the following numbers:

8.	.1236	.00061	.32760.
9.	.0080	.720000	.040721.
10.	71.03	240.01376	9.800000.

11. Write four hundred, and nine tenths. *Ans.* 400.9.
12. Write twenty-seven, and fifty-six hundredths.
13. Write eighty-five, and one thousandth.
14. Write one thousand, and twelve millionths.
15. Write three hundred sixty-five, and one thousand eight hundred seven hundred-thousandths.

REDUCTION.

EXAMPLES.

111. To reduce decimals to a common denominator.

1. Reduce .3, .09, .0426, .214 to a common denominator.

OPERATION.

.3000
.0900
.0426
.2140

SOLUTION. — A common denominator must contain as many decimal places as any of the given decimals. The third number contains four decimal places, and hence 10000 must be a common denominator. As annexing ciphers to decimals does not alter their value, we give to each number four decimal places, by annexing ciphers, and thus reduce the given decimals to a common denominator.

Rule. *Change all the numbers to the same decimal denomination by annexing ciphers when necessary.*

2. Reduce .7, .073, .42, .0020, and .007 to a common denominator.

3. Reduce .004, .00032, .6, .37, and .0314 to a common denominator.

4. Reduce 1 tenth, 46 hundredths, 15 thousandths, 462 ten-thousandths, and 28 hundred-thousandths to a common denominator.

5. Reduce 9 thousandths, 9 ten-thousandths, and 9 hundred-thousandths to a common denominator.

6. Reduce 42.07, 102.006, 7.80, 400.01234 to a common denominator.

7. Reduce 300.3, 8.1003, 14.12614, 210.000009, and 1000.02 to a common denominator.

8. Reduce 3450, .3450, 3.456, and 34.506 to a common denominator.

9. Reduce .2, .425, .6575, .34, and .35742 to a common denominator.

112. To reduce a decimal to a common fraction.

1. Reduce .125 to an equivalent common fraction.

OPERATION.

$.125 = \frac{125}{1000} = \frac{1}{8}$

SOLUTION. — Writing the figures of the decimal, 125, over the denominator 1000, we have $\frac{125}{1000} = \frac{1}{8}$.

Rule. *Omit the decimal point, supply the proper denominator, and then reduce the fraction to its lowest terms.*

Reduce to common fractions:

2. .08.	*Ans.* $\frac{2}{25}$.	5. .008.	*Ans.* $\frac{1}{125}$.
3. .625.	*Ans.* $\frac{5}{8}$.	6. .4.	*Ans.* $\frac{2}{5}$.
4. .375.	*Ans.* $\frac{3}{8}$.	7. .024.	*Ans.* $\frac{3}{125}$.

113. To reduce a common fraction to a decimal.

1. Reduce $\frac{3}{4}$ to its equivalent decimal.

OPERATION.

4)3.0 (7 tenths.
2.8
4)20 (5 hundredths.
20
Ans. .75.

or, 4)3.00
.75, *Ans.*

SOLUTION. — We reduce the numerator 3 to *tenths* by annexing a cipher, 3.0. Since $\frac{3}{4}$ (the fraction) is $\frac{1}{4}$ of 3, we divide the result, 3.0, by 4, and obtain 7 tenths, and a remainder of 2 tenths. Reducing this remainder to *hundredths*, by annexing a cipher, and dividing by 4, as before, we obtain 5 hundredths. The sum of the quotients is .75, *Ans.*

Rule. I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the result as are equal to the number of ciphers annexed.*

Reduce to decimals:

2. $\frac{1}{2}$.	*Ans.* .5.	6. $\frac{1}{25}$.	*Ans.* .04.
3. $\frac{2}{5}$.	*Ans.* .4.	7. $\frac{5}{16}$.	*Ans.* .3125.
4. $\frac{5}{8}$.	*Ans.* .625.	8. $\frac{17}{20}$.	*Ans.* .85.
5. $\frac{9}{10}$.	*Ans.* .9.	9. $\frac{2}{125}$.	*Ans.* .016.

ADDITION.

114. Since the same law of local value extends to the right and left of units' place, that is, since decimals and simple integers increase and decrease uniformly by the scale of ten, it is evident that decimals may be added, subtracted, multiplied, and divided in the same manner as integers.

EXAMPLES.

1. What is the sum of 4.314, 36.42, 120.0042, and .4276?

OPERATION.

4.314
36.42
120.0042
.4276
161.1658

SOLUTION. — We write the numbers so that the figures of like orders of units stand in the same columns; that is, units under units, tenths under tenths, hundredths under hundredths, etc. This brings the decimal points directly under one another. Commencing at the right hand, we add each column separately, and carry as in whole numbers. In the result, we place a decimal point between units and tenths, or directly under the decimal point in the numbers added.

Rule. I. *Write the numbers so that the decimal points stand directly under one another.*

II. *Add as in whole numbers, and place the decimal point in the result directly under the points in the numbers added.*

2. What is the sum of 2.7, 30.84, 75.1, 126.414, and 3.06? *Ans.* 238.114.

3. What is the sum of 1.7, 4.45, 6.75, 1.705, .50, and .05? *Ans.* 15.155.

4. Add 105.7, 19.4, 1119.05, 648.006, and 19.041. *Ans.* 1911.197.

5. Add 48.1, .0481, 4.81, .00481, 481. *Ans.* 533.96291.

6. Add 1.151, 13.29, 116.283, 9.0275, and .61.

Ans. 140.3615.

7. Add .8, .087, .626, .8885, and .49628.

8. What is the sum of 91.003, 16.4691, 160.00471, 700.05, 900.0006, .0315? *Ans.* 1867.55891.

9. What is the sum of fifty-four, and thirty-four hundredths; one, and nine ten-thousandths; three, and two hundred seven millionths; twenty-three thousandths; eight, and nine tenths; four, and one hundred thirty-five thousandths? *Ans.* 71.399107.

10. How many acres of land in four farms, containing respectively 61.843 acres, 120.75 acres, 142.4056 acres, and 180.750 acres? *Ans.* 505.7486.

11. How many yards of cloth in 3 pieces, the first containing $21\frac{1}{2}$ yards, the second $36\frac{3}{4}$ yards, and the third 40.15 yards? *Ans.* 98.40 yards.

12. A man owns 4 city lots, containing $32\frac{1}{4}$, $36\frac{3}{8}$, $40\frac{5}{8}$, 42.73 rods of land, respectively. How many rods does he own in all? *Ans.* 152.205 rods.

13. A man bought a house for $4500, a store for $18000, merchandise for $19753.75, and a horse for $189.80. What did the whole cost him?

NOTE. In expressions of United States currency, dollars may be regarded as whole numbers, and cents as decimal parts of a number.

14. Mr. Brown paid the following sums of money for repairs in his house: to John Smith $390.65, to Johnson & Gardner $492.57, to J. S. Green & Co. $49.25, and to Rufus & Coate $2340.75. How much did he pay for the repairs?

15. How many yards in three pieces of cloth, the first piece containing 18.375 yards, the second piece 41.625 yards, and the third piece 35.5 yards?

SUBTRACTION.

EXAMPLES.

115. 1. From 124.2750 take 47.3126.

OPERATION.

124.2750
47.3126
Ans. 76.9624

SOLUTION. — We write the subtrahend under the minuend, placing units under units, tenths under tenths, etc. Commencing at the right hand, we subtract as in whole numbers, and in the remainder place the decimal point directly under those in the numbers above. If the number of decimal places in the minuend and subtrahend are not equal, they may be reduced to the same number of decimal places before subtracting, by annexing ciphers.

Rule. I. *Write the numbers so that the decimal points stand directly under one another.*

II. *Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.*

2. From 463.05 take 17.0613. *Ans.* 445.9887.
3. From 134.63 take 101.1409. *Ans.* 33.4891.
4. From 189.6145 take 10.151. *Ans.* 179.4635.
5. From 671.617 take 116.1. *Ans.* 555.517.
6. From 480 take 245.0075. *Ans.* 234.9925.
7. Subtract .09684 from .145. *Ans.* .04816.
8. Subtract .2371 from .2754. *Ans.* .0383.
9. Subtract .0007 from 107. *Ans.* 106.9993.
10. Subtract $3\frac{3}{4}$ from 9.3261. *Ans.* 5.5761.
11. Subtract 25.072 from $112\frac{5}{8}$. *Ans.* 87.553.

12. A man owned fifty-four hundredths of a township of land, and sold fifty-four thousandths of the same. How much did he still own? *Ans.* .486.

13. A man owning 475 acres of land, sold at different times 80.75 acres, $100\frac{1}{8}$ acres, and 125.625 acres. How much land had he left? *Ans.* 168.5 acres.

MULTIPLICATION.

EXAMPLES.

116. 1. What is the product of .25 multiplied by .5?

OPERATION.

.25
.5
.125, *Ans.*

SOLUTION. — We first multiply as in whole numbers; then, since the multiplicand has 2 decimal places and the multiplier 1, we point off $2 + 1 = 3$ decimal places in the product. The reason for this will be evident, by considering both factors as common fractions, and then multiplying as in (**99**), thus: $.25 = \frac{25}{100}$, and $.5 = \frac{5}{10}$; and $\frac{25}{100} \times \frac{5}{10} = \frac{125}{1000}$, which written decimally is .125.

Rule. *Multiply as in whole numbers, and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply a decimal by 10, 100, 1000, remove the point as many places to the right as there are ciphers on the right of the multiplier.

2.	3.	4.
.241	9.4263	.01346
.7	.5	.06
.1687	4.71315	.0008076

5. Multiply 7.1 by 8.2. *Ans.* 58.22.
6. Multiply 15.5 by .08. *Ans.* 1.24.
7. Multiply 8.123 by .09. *Ans.* .73107.
8. Multiply 4.5 by .15. *Ans.* .675.
9. Multiply 450 by .02. *Ans.* 9.
10. Multiply 341.45 by .007. *Ans.* 2.39015.
11. Multiply 3020 by .015. *Ans.* 45.3.
12. Multiply .132 by .241. *Ans.* .031812.
13. Multiply .23 by .009. *Ans.* .00207.
14. Multiply 7.02 by 5.27. *Ans.* 36.9954.

15. Multiply .004 by .04. *Ans.* .00016.

16. Multiply 2461 by .0529. *Ans.* 130.1869.

17. Multiply .007853 by .035. *Ans.* .000274855.

18. Multiply 25.238 by 12.17. *Ans.* 307.14646.

19. Multiply .3272 by 10. *Ans.* 3.272.

20. Multiply .3272 by 100. *Ans.* 32.72.

21. Multiply .3272 by 1000. *Ans.* 327.2.

22. Find the value of $.25 \times .5 \times 12$. *Ans.* 1.5.

23. Find the value of $.07 \times 2.4 \times .015$. *Ans.* .00252.

24. Find the value of $6\frac{1}{2} \times .8 \times 3.16$. *Ans.* 16.432.

25. If a man travels 3.75 miles an hour, how far will he travel in 9.5 hours? *Ans.* 35.625 miles.

26. If a sack of salt contains 94.16 pounds, how many pounds will 17 such sacks contain? *Ans.* 1600.72 pounds.

27. If a man spends .87 of a dollar in 1 day, what will he spend in 15.525 days? *Ans.* $13.50675.

28. One rod is equal to 16.5 feet. How many feet are there in 30.005 rods? *Ans.* 495.0825 feet.

29. How many gallons of molasses are there in .54 of a barrel, there being 31.5 gallons in 1 barrel?

30. Multiply 25 by .25. *Ans.* 6.25.

31. Multiply .132 by .241. *Ans.* .031812.

32. Multiply 24.35 by 10.

33. Multiply .006 by 1000. *Ans.* 6.

34. Multiply .23 by .009. *Ans.* .00207.

35. Multiply sixty-four thousandths by thirteen millionths. *Ans.* .000000832.

36. Multiply eighty-seven ten-thousandths by three hundred fifty-two hundred-thousandths.

37. Multiply sixteen thousand by sixteen ten-thousandths. *Ans.* 25.6.

38. If a cord of wood is worth 2.37 bushels of wheat, how many bushels of wheat must be given for 9.58 cords of wood? *Ans.* 22.7046 bushels.

DIVISION.

EXAMPLES.

117. 1. What is the quotient of .225 divided by .5?

OPERATION.

.5).225

.45, *Ans.*

SOLUTION. — We perform the division in the same manner as in whole numbers, and the only difficulty we meet with is in pointing off the decimal places in the quotient. To determine how many places to point off, we reduce the decimals to common fractions, thus: $.225 = \frac{225}{1000}$ and $.5 = \frac{5}{10}$; performing the division as in (**97**), we have $\frac{225}{1000} \div \frac{5}{10} = \frac{225}{1000} \times \frac{10}{5} = \frac{45}{100} = .45$. Here we see that the dividend contains as many decimal places as are contained in both divisor and quotient.

Rule. *Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.*

1. If the number of figures in the quotient is less than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by prefixing ciphers.

2. If there is a remainder after dividing the dividend, annex ciphers, and continue the division; the ciphers annexed are decimals of the dividend.

3. The dividend must always contain at least as many decimal places as the divisor, before commencing the division.

4. To divide by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers on the right hand of the divisor.

2.	3.	4.	5.
.6).426	.8)3.7624	.05)81.60	.009).00207
.71.	4.703	1632	.23

6.	7.	8.
.075).9375(12.5	.288)18.0000(62.5	.0025)15.8750(6350
75	1728	150
187	720	87
150	576	75
375	1440	125
375	1440	125
		0
		0

9. Divide 44 by .4. *Ans.* 110.
10. Divide 15 by .25. *Ans.* 60.
11. Divide .3276 by .42. *Ans.* .78.
12. Divide .00288 by .08. *Ans.* .036.
13. Divide .0992 by .32. *Ans.* .31.
14. Divide 17.6 by 44. *Ans.* .4.
15. Divide .0000021 by .0007. *Ans.* .003.
16. Divide .56 by 1.12. *Ans.* .5.
17. Divide 1496.04 by 10. *Ans.* 149.604.
18. Divide 1496.04 by 100. *Ans.* 14.9604.
19. Divide 1596.04 by 1000. *Ans.* 1.59604.
20. Divide 4.96 by 100. *Ans.* .0496.
21. Divide 10 by .1. *Ans.* 100.
22. Divide 100 by .2. *Ans.* 500.
23. If 2.5 acres produce 34.75 bushels of wheat, how much does one acre produce? *Ans.* 13.9 bushels.
24. If a man travels 21.4 miles a day, how many days will he require to travel 461.03 miles?
25. If a man builds 812.5 rods of fence in 100 days, how many rods does he build each day?
26. Paid $131.15 for 61 sheep. What was paid for each? *Ans.* $2.15.

PROMISCUOUS EXAMPLES.

1. Add twenty-five hundredths, six hundred fifty-four thousandths, one hundred ninety-nine thousandths, and seven thousand five hundred sixty-nine ten-thousandths. *Ans.* 1.8599.
2. From ten take ten thousandths. *Ans.* 9.99.
3. What is the difference between forty thousand, and forty thousandths? *Ans.* 39999.960.
4. Multiply sixty-five hundredths, by nine hundredths. *Ans.* .0585.
5. Divide 324 by 6400. *Ans.* .050625.

6. Reduce .125 to a common fraction. *Ans.* $\frac{1}{8}$.

7. Reduce $\frac{7}{8}$ to a decimal fraction. *Ans.* .875.

8. Divide .016004 by .004. *Ans.* 4.001.

9. Reduce $\frac{17}{25}$ to a decimal fraction. *Ans.* .68.

10. Reduce .4, .007, .1142, .036, .00015, and .42, to a common denominator.

11. At 13.9 dollars a ton, what will 2.5 tons of hay cost? *Ans.* 34.75 dollars.

12. If a pound of sugar costs \$.09, how many pounds can be bought for \$5.85? *Ans.* 65 pounds.

13. If 40.02 bushels of potatoes are raised upon 1 acre of land, how many acres will be required to raise 4580.64 bushels? *Ans.* 114.458 acres.

14. At \$11 a ton, how much hay can be bought for \$13.75? *Ans.* 1.25 tons.

15. If a man travels 32.445 miles in a day, how far can he travel in .625 of a day? *Ans.* 20.278125 miles.

16. If 2 pounds of sugar cost \$.1875, what will be the cost of 10 pounds? *Ans.* \$.9375.

17. If 3 barrels of apples cost \$19.125, what will be the cost of 100 barrels? *Ans.* \$637.50.

18. What is the difference between nine million and nine millionths? *Ans.* 8999999.999991.

19. If one acre produces 42.57 bushels of corn, how many bushels will 18.73 acres produce? *Ans.* 797.3361.

20. If 6.35 acres produce 70.6755 bushels of wheat, what does one acre produce? *Ans.* 11.13 bushels.

21. How many times will .5 of 1.75 be contained in .25 of $17\frac{1}{2}$? *Ans.* 5.

22. What will be the cost of $3\frac{5}{8}$ bales of cloth, each bale containing 36.75 yards, at .85 dollars per yard?

23. Traveling at the rate of $4\frac{3}{5}$ miles an hour, how many hours will a man require to travel 56.925 miles?

Ans. $12\frac{3}{8}$ hours.

UNITED STATES MONEY.

118. United States Money is the legal currency of the United States, and was established by act of Congress, August 8, 1786. Its denominations and their relative values are shown in the following table:

TABLE.

10 Cents	make	1 Dime	d.
10 Dimes	"	1 Dollar	$.
10 Dollars	"	1 Eagle	E.

The currency of the United States is *decimal* currency, and is sometimes called *Federal Money*.

119. The character $ before any number indicates that it expresses United States money.

Thus, $75 expresses 75 dollars.

The dollar is the *unit* of United States money; dimes and cents are fractions of a dollar, and are separated from the dollar by the *decimal point* (.).

Thus, two dollars one dime two cents are written, $2.12.

120. By examining the above table we find that

1. The dollar being the unit, dimes and cents are respectively tenths and hundredths of a dollar.

2. The denominations of United States money increase and decrease in the same way as simple numbers and decimals, and are expressed according to the decimal system of notation.

Hence we see that United States money may be added, subtracted, multiplied, and divided in the same manner as decimals.

Dimes are not read as *dimes*, but the two places of dimes and cents are appropriated to *cents*.

Thus 1 dollar 3 dimes 2 cents, or $1.32, is read *one dollar thirty-two cents.*

When the number of cents is less than 10, we *write a cipher in the place of dimes.*

121. COINS. — The *gold coins* are the double eagle, the eagle, the half-eagle, and the quarter-eagle.

The *silver coins* are the dollar, the half-dollar, the quarter-dollar, and the ten-cent piece.

The *nickel coin* is the five-cent piece.

The *bronze coin* is the one-cent piece.

EXAMPLES.

1. Write four dollars eighteen cents. *Ans.* $4.18.
2. Write five dollars twenty-five cents.
3. Write twelve dollars thirty-six cents.
4. Write seven dollars sixteen cents.
5. Write ten dollars ten cents.
6. Write sixty-five cents. *Ans.* $.65.
7. Write one dollar five cents. *Ans.* $1.05.
8. Write eighty-seven cents. *Ans.* $.87.
9. Write one hundred dollars one cent. *Ans.* $100.01.
10. Read $4.07; $3.09; $10.50; $25.02.
11. Read $35.19; $6.45; $143.00; $9.22.

REDUCTION.

EXAMPLES.

122. To change dollars to cents.

1. How many cents are there in 75 dollars?

OPERATION.

75
100
7500 cents.

SOLUTION. — Since in 1 dollar there are 100 cents, in 75 dollars there are 75 times 100 cents or 7500 cents.

Rule. *To change dollars to cents, multiply by 100; that is, annex two ciphers.*

Reduce to cents:

2. $24. *Ans.* 2400 cents.
3. $42. *Ans.* 4200 cents.
4. $14. *Ans.* 1400 cents.
5. $102. *Ans.* 10200 cents.
6. $455. *Ans.* 45500 cents.
7. $35.
8. $66.
9. $73.
10. $4.28.
11. $18.07.

To change cents to dollars.

1. In 3427 cents how many dollars?

OPERATION.

1|00)34|27
$34.27, *Ans.*

SOLUTION. — Since 100 cents equal 1 dollar, 3427 cents equal as many dollars as 100 is contained times in 3427, which is 34.27 times. Therefore 3427 cents = $34.27.

Rule. *To change cents to dollars, divide by 100; that is, point off two figures from the right.*

Reduce to dollars:

2. 972 cents. *Ans.* $9.72.
3. 1609 cents. *Ans.* $16.09.
4. 3476 cents. *Ans.* $34.76.
5. 34671 cents. *Ans.* $346.71.
6. 10307 cents. *Ans.* $103.07.
7. 203062 cents.
8. 672 cents.
9. 3104 cents.
10. 17826 cents.
11. 34325 cents.

ADDITION.

EXAMPLES.

123. 1. What is the sum of $12.50, $8.12, $4.07, $15.37, and $22?

OPERATION.

$12.50
8.12
4.07
15.37
22.00

$62.06, *Ans.*

SOLUTION. — Writing dollars under dollars, cents under cents, etc., so that the decimal points stand under each other, we add and point off as in addition of decimals.

Rule. I. *Write dollars under dollars, cents under cents.*

II. *Add as in simple numbers, and place the point in the amount as in addition of decimals.*

Add:

2.	3.	4.	5.
$126.08	$100.37	$750.00	$1042.87
42.64	13.09	140.07	427.03
304.12	65.82	35.17	50.50
14.42	400.00	6.00	7.08

6.	7.	8.	9.
$340.32	$99.14	$4320.00	$316.00
96.17	2.79	9.99	42.98
17.96	.63	550.09	31.76
430.22	15.34	650.00	513.80

10. What is the sum of 30 dollars 9 cents, 200 dollars 63 cents, 27 dollars 36 cents, and 10 dollars 16 cents? *Ans.* $268.24.

11. A grocer bought a barrel of sugar for $17.84, a box of tea for $36.12, a cheese for $4, and a tub of butter for $7.09. What was the cost of all?

12. A merchant bought a quantity of goods for $458.25; paid for duties $45, for freights $98.62, and for insurance $16.40. What was the whole cost?

Ans. $618.27.

13. Bought some sugar for $1.75, some tea for $.90, some butter for $2.12, some eggs for $.37, and some spice for $.25. What was the cost of the whole? *Ans.* $5.39.

14. Paid for building a house $1045.75, for painting the same $275.60, for furniture $648.87, and for carpets $105.10. What was the cost of the house and furnishing? *Ans.* $2075.32.

15. A farmer receives $120.45 for wheat, $36.62 for corn, $14.09 for potatoes, $63.00 for oats, and $75.00 for butter. How much does he receive for the whole?

16. A lady who went shopping, bought a dress for $7.27, trimmings for $.87, some tape for $.06, some thread for $.12, and some needles for $.09. What did she pay for all? *Ans.* $8.41.

17. I paid $390.37 for some furniture, $187.50 for a piano, $90.05 for a month's rent, and $400.00 for a carriage and horses. How much did I pay for all?

18. A lady paid $45.40 for some furs, $12.37 for a dress, $5 for a bonnet, and $1.12 for a pair of gloves. What did she pay for all?

19. A farmer sold a cow for $20, a horse for $96 50, a yoke of oxen for $66.87, and a ton of hay for $9.40. What did he receive for all? *Ans.* $192.77.

20. I bought a hat for $4.50, a pair of boots for $5.62, an umbrella for $2.12, and a pair of gloves for $.87. What was the cost of the whole? *Ans.* $13.11.

SUBTRACTION.

EXAMPLES.

124. 1. From $246.82 take $175.27.

OPERATION.

$246.82
175.27
$71.55, *Ans.*

SOLUTION. — Writing the less number under the greater, dollars under dollars, cents under cents, etc., we subtract and point off in the result as in subtraction of decimals.

Rule. I. *Write the subtrahend under the minuend, dollars under dollars, cents under cents.*

II. *Subtract as in simple numbers, and place the point in the remainder as in subtraction of decimals.*

	2.	3.	4.	5.
From	$125.05	$327.10	$112.00	$43.37
Take	43.27	100.09	.87	2.06
Ans.	$81.78	$227.01	$111.13	$41.31

6. From $3472.50 take $1042.12. *Ans.* $2430.38.
7. From $540 take $256.67. *Ans.* $283.33.
8. From $82.04 take $80.62. *Ans.* $1.42.
9. From 3 dollars 10 cents take 75 cents. *Ans.* $2.35.
10. From 10 dollars take 5 dollars 10 cents.
11. From 100 dollars take 50 dollars 50 cents.
12. From 1001 dollars 9 cents take 300 dollars. *Ans.* $701.09.
13. From 2 dollars take 75 cents. *Ans.* $1.25.
14. From 96 cents take 12 cents. *Ans.* $.84.
15. From 1 dollar take 25 cents. *Ans.* $.75.
16. From 50 cents take 37 cents. *Ans.* $.13.
17. From 5 dollars take 50 cents. *Ans.* $4.50.
18 From 4 dollars take 1 dollar 40 cents.

19. I sold a horse for $200, which was $45.50 more than it cost me. What did it cost me? *Ans.* $154.50.

20. A man bought a farm for $4640, and sold it for $5027.50. What did he gain? *Ans.* $387.50.

21. A man borrowed $25, and returned $15.60. How much remained unpaid? *Ans.* $9.40.

22. A merchant having $10475, paid $2426 for a store, and $5327.87 for goods. How much money had he left? *Ans.* $2721.13.

23. I bought a sack of flour for $3.12. How much change must I receive from a $5 bill? *Ans.* $1.88.

24. I bought groceries to the amount of $1.87. How much change must I receive from a $2 bill?

Ans. $.13.

25. I paid $375 for a pair of horses; one of them cost $215.50. What did the other one cost me?

Ans. $159.50.

26. I started on a journey with $50, and paid $10.62 railroad fare, $7.38 stage fare, $5.96 for board and lodging, and $.75 for expressage. How much money had I left? *Ans.* $25.29.

27. A farmer sold some wool for $27.16, and a ton of hay for $14.80. He received in payment a barrel of flour worth $6.87, and the remainder in money. How much money did he receive? *Ans.* $35.09.

28. A woman sold a grocer some butter for $1.48, and some eggs for $.94. She received a gallon of molasses worth $.40, a pound of tea worth $.75, and a pound of starch worth $.12. How much is still due her?

Ans. $1.15.

29. A tailor bought a piece of broadcloth for $87.50, and a piece of cassimere for $62.75. He sold both pieces for $170.87. What did he gain on both?

Ans. $20.62

MULTIPLICATION.

EXAMPLES.

125. 1. Multiply $26.14 by 34.

OPERATION.

$26.14
34
10456
7842
$888.76, *Ans.*

SOLUTION. — We multiply as in simple numbers, always regarding the multiplier as an *abstract* number, and point off from the right hand of the result, as in multiplication of decimals.

Rule. *Multiply as in simple numbers and place the point in the product as in multiplication of decimals.*

2.	3.	4.	5.
$327.48	$82.37	$160.09	$97.87
15	46	87	123

6. What do 8 cords of wood cost at $3.50 a cord?
Ans. $28.

7. What is the cost of 14 barrels of flour, at $5.85 a barrel?

8. What is the cost of 25 bushels of corn, at $.75 a bushel?

9. At $2.12 a yard, what will 18 yards of silk cost?

10. At $.87 apiece, what will be the cost of 9 turkeys?

11. A farmer sold 40 bushels of potatoes at $.37 a bushel, and 21 barrels of apples at $2.25 a barrel. What did he receive for both? *Ans.* $62.05.

12. Bought 124 acres of land at $35.75 an acre, and sold the whole for $6000. What did I gain?

13. What will be the cost of 275 bushels of oats, at $.42 a bushel? *Ans.* $115.50.

14. What will be the cost of 15 yards of broadcloth, at $4.87 a yard? *Ans.* $73.05.

DIVISION.

EXAMPLES.

126. 1. Divide $136 by 64.

OPERATION.

64)$136.00($2.12½, *Ans.*
128
80
64
160
128
32

SOLUTION. — We divide as in simple numbers, and as there is a remainder after dividing the dollars, we reduce the dividend to cents and a fraction thereof by annexing two ciphers, and continuing the division.

Rule. *Divide as in simple numbers, and place the point in the quotient, as in division of decimals.*

In business transactions it is never necessary to carry the division further than to cents in the quotient. Ordinarily, if a fraction of a cent greater than ½ remains, one cent is added; if less than ½, no account is taken of the fraction.

2.	3.	4.	5.
5)$43.50	10)$36.00	8)$371	12)$169.50
$8.70	$3.60	$46.375	$14.125

6. Divide $13.75 by 11. *Ans.* $1.25.
7. Divide $162 by 36. *Ans.* $4.50.
8. Divide $246.30 by 15. *Ans.* $16.42.
9. Divide $1305 by 18. *Ans.* $72.50.
10. Paid $168.48 for 144 bushels of wheat. What was the price per bushel? *Ans.* $1.17.
11. Paid $2.80 for 35 pounds of sugar. What was the price per pound? *Ans.* $.08.
12. If 54 cords of wood cost $135, what is the price per cord? *Ans.* $2.50.
13. Bought 125 bushels of oats for $62.50. What was the cost per bushel? *Ans.* $.50.
14. If 70 barrels of apples cost $175, what will 1 barrel cost? *Ans.* $2.50.

PROMISCUOUS EXAMPLES.

1. A merchant bought 14 boxes of tea for $560; but as it was damaged, he was obliged to sell it for $106.75 less than it cost him. What did he receive per box?

Ans. $32.37½.

2. A farmer sold 120 bushels of wheat, at $1.12½ a bushel, and received in payment 27 barrels of flour. What did the flour cost him per barrel?

3. If 35 yards of cloth cost $122.50, what will 29 yards cost? *Ans.* $101.50.

4. If 4 tons of coal cost $35.50, what will 12 tons cost? *Ans.* $106.50.

5. If 29 pounds of sugar cost $3.62½, what will 15 pounds cost? *Ans.* $1.87½.

6. If 12 barrels of flour cost $108, what will 18 barrels cost? *Ans.* $162.

7. If 3 bushels of wheat cost $4.35, what will 30 bushels cost? *Ans.* $43.50.

8. A man bought a farm containing 125 acres, for $2922.50. For what must he sell it per acre to gain $500? *Ans.* $27.38.

9. A farmer exchanged 50 bushels of corn worth 70 cents a bushel, for 28 bushels of wheat. What was the wheat worth a bushel? *Ans.* $1.25.

10. A man having $15000, bought 30 bales of cotton, each bale containing 940 pounds at $.10 a pound; he next paid $6680 for a house, and then bought 1000 barrels of flour with the money he had left. What did the flour cost him per barrel? *Ans.* $5.50.

11. A merchant bought goods to the amount of $7425.50; he paid for duties on the same $253.96, and for freight $170.09. What was the entire cost of the goods? *Ans.* $7849.55.

BILLS.

127. A Bill, in business transactions, is a written statement of articles bought or sold, together with the price of each, and the whole cost.

Find the cost of the several articles, and the amount or footing of the following bills:

1.

CHICAGO, Sept. 20, 1891.

MR. J. C. SMITH,

Bought of SILAS JOHNSON,

36 pounds sugar at $.04 a pound,	$1.44
18 pounds coffee at $.25 a pound,	4.50
24 pounds butter at $.30 a pound,	7.20
10 dozen eggs at $.18 a dozen,	1.80
4 gallons molasses at $.44 a gallon,	1.76
Ans.	$16.70

2.

ROCHESTER, Jan. 25, 1892.

JOHN DABNEY, ESQ.,

Bought of BARDWELL & Co.,

14 pounds coffee at $.25 a pound,	$3.50
6 pounds tea at $.80 a pound,	4.80
25 pounds No. 1 mackerel at $.06 a pound,	1.50
5 bushels potatoes at $.38 a bushel,	1.90
3 gallons sirup at $.80 a gallon,	2.40
7 dozen eggs at $.16 a dozen,	1.12
Ans.	$15.22

Received Payment,

BARDWELL & Co.,

per Adams.

3.

MEMPHIS, Aug. 20, 1892.

MR. S. P. HAILE,

Bought of PATTERSON & Co.,

20 chests	Green Tea	at	$22.50
16 "	Black "	"	18.75
14 "	Imperial "	"	32.87½
15 sacks	Java Coffee	"	17.38
25 boxes	Oranges	"	4.62½
			$1586.57½

Received Payment,

PATTERSON & Co.

4.

OSWEGO, Sept. 4, 1891.

JAMES COROVAL & Co.,

Bought of COLLINS & SON,

12 yards	Broadcloth	at	$3.84
18 "	Cassimere	"	2.25
10 "	Satinet	"	.87½
42 "	Flannel	"	.45
35 "	Black Silk	"	1.18
			$155.53

5.

BOSTON, April 10, 1892.

J. G. BENNET & SON,

Bought of BUTLER, KING & Co.,

14	Plows	at	$10.50
8	Harrows	"	9.80
120	Shovels	"	.90
175	Hoes	"	.62½
			$442.77½

DENOMINATE NUMBERS.

128. A concrete number in which the unit of measure is established by law or custom is called a **Denominate Number.**

Thus, 3 cents, 4 pounds, 5 hours are denominate numbers.

129. Denominate Numbers may be **Simple** or **Compound.**

A *Simple Denominate Number* is one which is composed only of units of the same denomination.

Thus, 5 hours, 4 yards, 3 quarts are simple denominate numbers.

A *Compound Denominate Number* is one whose value is expressed in two or more different denominations.

Thus, 32 dollars 15 cents; 15 days 4 hours 25 minutes.

130. A **Scale** is a series of decreasing or increasing numbers.

In simple numbers and decimals the scale is uniformly 10; in compound numbers the scales vary.

CURRENCY.

131. I. United States Money.

For table of United States money see **118.**

II. Canada Money.

132. The currency of the Dominion of Canada is decimal, and the table and denominations are the same as those of United States money.

The currency of the Dominion of Canada was made uniform July 1st, 1871. Before the adoption of the decimal system, pounds, shillings, and pence were used.

Coins. — The Canadian coins are silver and bronze.

The *silver coins* are the fifty-cent piece, the twenty-five-cent piece, the ten-cent piece, and the five-cent piece.

The *bronze coin* is the cent.

The *gold coins* used in Canada are the *British sovereign*, worth $4.86⅔, and the *half-sovereign*.

III. English Money.

133. English or Sterling money is the currency of Great Britain.

Table.

4 Farthings (far.)	make	1 Penny	*d.*
12 Pence	"	1 Shilling	*s.*
20 Shillings	"	1 Pound or Sovereign .	£ or sov.

Farthings are generally expressed as fractions of a penny; sometimes called 1 quarter (qr.) = ¼ *d.*; 3 far. = ¾ *d.*

Coins. — The *gold coins* are the sovereign (= £1) and the half-sovereign (= 10 *s.*).

The *silver coins* are the crown (5 *shillings*), half-crown, florin (2 *shillings*), shilling, sixpenny, fourpenny, and threepenny pieces.

The *copper* and *bronze coins* are the penny, halfpenny, and farthing.

1. How many pence in 3 *s.* 6 *d.*?
2. How many crowns make a pound?

3. How many pounds in 100 shillings? How many crowns? How many farthings?

4. At two florins a yard, how many yards of cloth will three sovereigns purchase?

5. When oranges are worth threepence each, how many can you buy for a florin?

6. How many yards of ribbon, at sixpence a yard, can be purchased for 2 shillings?

7. If one book costs £1 12*s.*, what will ten books of the same kind cost?

MEASURES OF WEIGHT.

134. Weight is a measure of the quantity of matter a body contains, determined according to some fixed standard.

I. Troy Weight.

135. Troy Weight is used in weighing gold, silver, and jewels in philosophical experiments, etc.

Table.

24 Grains (gr.)	make	1 Pennyweight . . .	pwt. or dwt.
20 Pennyweights	"	1 Ounce	oz.
12 Ounces	"	1 Pound	lb.

II. Avoirdupois Weight.

136. Avoirdupois Weight is used for all the ordinary purposes of weighing.

Table.

16 Ounces (oz.)	make	1 Pound	lb.
100 Pounds	"	1 Hundred-weight	cwt.
20 Cwt. = 2000 lb.	"	1 Ton	T.

137. The **Metric System** of weights and measures is based upon a decimal scale. Metric weights and measures were authorized by Congress in 1866.

Metric Measure of Weight.

138. The **Gram** is the *unit of weight*, and equal to the weight of a cube of distilled water, the edge of which is *one hundredth* of a *meter*, equal to 15.432 Troy grains.

Table.

10 Centigrams, *cg.* = 1 Decigram.
10 Decigrams, *dg.* = 1 **Gram.**
10 Grams, *G.* = 1 Dekagram.
10 Dekagrams, *Dg.* = 1 Hectogram.
10 Hectograms, *Hg.* = 1 Kilogram, or **Kilo.**

The *Kilogram*, or *Kilo*, is the *unit* of common weight in trade, and is a trifle less than $2\frac{1}{5}$ lb. Avoirdupois.

1. How many ten-grain powders can be made from an ounce of quinine?

2. When coal is worth $6 a ton, how much will 5 cwt. cost?

3. How many kilograms in 11 pounds Avoirdupois?

4. If a grocer purchases 10 kilograms of prunes at 33 cents per kilo, and sells them at 20 cents per pound, how much does he gain?

MEASURES OF EXTENSION.

139. **Extension** has three dimensions — *length*, *breadth*, and *thickness*.

A **Line** has only one dimension — *length*. A **Surface** or **Area** has two dimensions — *length* and *breadth*. A **Solid** or **Body** has three dimensions — *length*, *breadth*, and *thickness*.

I. Long Measure.

140. Long Measure, also called Linear Measure, is used in measuring lines or distances.

Table.

12	Inches (in.)	make	1	Foot	ft.
3	Feet	"	1	Yard	yd.
5½	Yards, or 16½ ft.,	"	1	Rod	rd.
320	Rods	"	1	Statute Mile	mi.

For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into *halves, quarters, fourths, eighths,* and *sixteenths.* The old table of Cloth Measure is practically obsolete.

Surveyors' Long Measure.

141. A **Gunter's Chain,** used by land surveyors, is 4 rods, or 66 feet long, and consists of 100 links.

Table.

7.92	Inches (in.)	make	1	Link	l.
25	Links	"	1	Rod	rd.
4	Rods, or 66 feet,	"	1	Chain	ch.
80	Chains	"	1	Mile	mi.

Metric Long Measure.

142. The **Meter** is the *unit of length,* and is equal to nearly 39.37 in.

Table.

Metric Denominations.

10	Centimeters,	*cm.* = 1		Decimeter.
10	Decimeters,	*dm.* = 1		**Meter.**
10	Meters,	*M.* = 1		Dekameter.
10	Dekameters,	*Dm.* = 1		Hectometer.
10	Hectometers,	*Hm.* = 1		Kilometer.
10	Kilometers,	*Km.* = 1		Myriameter (*Mm.*).

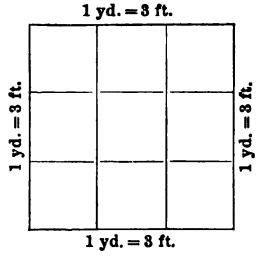
The *Meter*, like our yard, is used in measuring cloths and short distances.

The *Kilometer* is commonly used for measuring long distances, and is about $\frac{5}{8}$ of a common mile.

1. How many rods are there in 3 miles?
2. How many kilometers are there in 10 miles?
3. How many rods are there in 100 links?
4. If a train runs 40 kilometers each hour, how many miles does it run in 4 hours?
5. How many rods are there in half a mile?

II. Square Measure.

143. A **Square** is a figure having four equal sides, and four equal angles or corners.



1 square yard is a figure having four sides of 1 yard or 3 feet each, as shown in the diagram. Its contents are $3 \times 3 = 9$ square feet.

Thus, a square foot is 12 inches long, and 12 inches wide, and the contents are $12 \times 12 = 144$ square inches. A surface 20 feet long and 10 feet wide, is a rectangle, containing $20 \times 10 = 200$ square feet.

Rule. *To find the contents or area of a square, or of any other figure having a uniform length and a uniform breadth, multiply the length by the breadth.*

Square Measure is used in computing areas or surfaces; as of land, boards, painting, plastering, paving, etc.

TABLE.

144	Square Inches (sq. in.)	make	1 Square Foot	sq. ft.
9	Square Feet	"	1 Square Yard	sq. yd.
30¼	Square Yards	"	1 Square Rod	sq. rd.
160	Square Rods	"	1 Acre	A.
640	Acres	"	1 Square Mile	sq. mi.

SURVEYORS' SQUARE MEASURE.

144. This measure is used by surveyors in computing the area or contents of land.

TABLE.

625	Square Links (sq. l.)	make 1 Pole	P.
16	Poles	" 1 Square Chain .	sq. ch.
10	Square Chains	" 1 Acre	A.
640	Acres	" 1 Square Mile . .	sq. mi.
36	Square Miles (6 miles square)	" 1 Township	Tp.

1. A square mile of land is also called a *section*.

2. Canal and railroad engineers commonly use an engineer's chain, or a measuring tape, 100 feet long. A measuring tape is generally used in laying out city lots.

METRIC SQUARE MEASURE.

145. The **Are** is the *unit of land measure*, and is a square whose side is 10 meters, equal to a *square dekameter*, or 119.6 square yards.

TABLE.

1 Centare,	*ca.*	= 1 Square Meter.
100 Centares,	"	= 1 **Are.**
100 ARES,	*A.*	= 1 Hectare (*Ha.*).
100 Hectares,	*Ha.*	= 1 Square Kilometer.

The *square meter* is used for measuring such surfaces as are usually measured by the square yard.

Artificers estimate their work as follows:

By the square foot: glazing and stone-cutting.

By the square foot, or the square yard: painting, plastering, paving, ceiling.

By the square of 100 feet: flooring, partitioning, roofing, slating, and tiling.

Brick-laying is estimated by the thousand bricks; also by the square yard, and the square of 100 feet.

1. In estimating the painting of moldings, cornices, etc., the measuring line is carried into all the moldings and cornices.

2. In estimating brick-laying by the square yard or the square of 100 feet, the work is understood to be $1\frac{1}{2}$ bricks, or 12 inches, thick.

1. How many square yards are there in a ceiling 15 feet long and 12 feet wide?

2. How many acres are there in 3200 square rods?

3. What is the cost of 3 centares of plastering at 17 cents a square meter?

4. What is the cost of a quarter section of land, at $2.50 per acre?

5. Sixty-four men purchase a township, dividing the land equally among them. How much does each man own, and how much does it cost him at government price, $1.25 per acre?

6. How many poles are there in a township of land?

7. Reduce 3686400 P. to sq. mi.

8. In 94 A. 7 sq. ch. 12 P. 118 sq. l., how many square links are there?

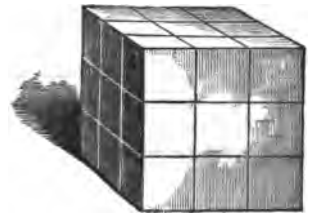
9. What will be the cost of a farm containing 4550000 square links, at $50 per acre? *Ans.* $2275.

10. How many square miles are there in 300 square kilometers?

11. How many centares are there in 5 hectares?

III. Cubic Measure.

146. A **Cube** is a solid, or body, having six equal square sides or faces.



If each side of a cube is 1 yard, or 3 feet, 1 foot in thickness of this cube will contain $3 \times 3 \times 1 = 9$ cubic feet; and the whole cube will contain $3 \times 3 \times 3 = 27$ cubic feet.

A solid, or body, may have the three dimensions all alike, or all different. A body 4 ft. long, 3 ft. wide, and 2 ft. thick contains $4 \times 3 \times 2 = 24$ cubic or solid feet.

Rule. *To find the cubic or solid contents of a body, multiply the length by the breadth, and the result by the thickness.*

Cubic Measure, also called Solid Measure, is used in estimating the contents of solids, or bodies; as timber, wood, stone, etc.

Table.

1728 Cubic Inches (cu. in.)	make	1 Cubic Foot . . cu. ft.
27 Cubic Feet	"	1 Cubic Yard . . cu. yd.
40 Cubic Feet of round timber, or 50 " " " hewn "	"	1 Ton or Load . . . T.
16 Cubic Feet	"	1 Cord Foot . . cd. ft.
8 Cord Feet, or 128 Cubic Feet	"	1 Cord of Wood . . Cd.
24¾ Cubic Feet	"	1 Perch of Stone, or Masonry . . Pch.

1. A cubic yard of earth is called a *load*.
2. Railroad and transportation companies estimate light freight by the space it occupies in cubic feet, and heavy freight by weight.
3. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord; and a cord foot is one foot in length of such a pile.
4. A perch of stone, or of masonry, is 16½ feet long, 1½ feet wide, and 1 foot high.
5. Embankments and excavations are estimated by the cubic yard.

Metric Cubic Measure.

147. The **Stere** is the *unit* of wood or solid measure, and is equal to a *cubic meter*, or .2759 cord.

Table.

10 Decisteres, *dst.* = 1 **Stere.**
10 Steres, *St.* = 1 Dekastere (*DSt.*).

The *Cubic Meter* is the *unit* for measuring ordinary solids; as excavations, embankments, etc.

1. How many loads of gravel will be needed to fill an excavation 36 ft. long, 20 ft. wide, and 6 ft. deep?
2. How many cords of wood are there in a pile 48 ft. long, 4 ft. wide, and 68 ft. high?
3. How many cubic feet are there in 20 tons of hewn timber?
4. How many steres are there in an embankment 40 meters long, 25 meters wide, and 5 meters high?

MEASURES OF CAPACITY.

148. Capacity signifies extent of room or space.

All measures of capacity are cubic measures, solidity and capacity being referred to different units, as will be seen by comparing the tables.

Measures of capacity may be properly subdivided into two classes: Measures of Liquids, and Measures of Dry Substances.

I. Liquid Measure.

149. Liquid Measure, also called Wine Measure, is used in measuring liquids; as liquors, molasses, water, etc.

Table.

4	Gills (gi.)	make 1	Pint	pt.
2	Pints	" 1	Quart	qt.
4	Quarts	" 1	Gallon	gal.
31½	Gallons	" 1	Barrel	bbl.
2	Barrels	" 1	Hogshead	hhd.
63	Gallons	" 1	Hogshead	hhd.

150. The following denominations are also in use in liquid measure:

36	Gallons	make 1	Barrel	of beer.
54	"	" 1	Hogshead	" "
1½	Barrels	" 1	"	" "
42	Gallons	" 1	Tierce.	
2	Hogsheads	" 1	Pipe, or Butt.	
2	Pipes, or 4 Hogsheads,	" 1	Tun.	

1. A bushel is commonly estimated at 2150.4 cubic inches.

2. The half-peck, or *dry gallon*, contains 268.8 cubic inches.

3. The standard liquid gallon contains 231 cu. in., equal to about 8⅓ lb. Avoir. of pure water.

4. Six *dry* gallons are equal to *seven liquid* gallons.

5. A *cubic foot* of pure water weighs 1000 oz., and equals 62½ lb. Avoir.

6. The denominations, barrel and hogshead, are used in estimating the capacity of cisterns, reservoirs, vats, etc.

7. The tierce, hogshead, pipe, butt, and tun are the names of *casks*, and do not express any fixed or definite measures. They are usually gauged, and have their capacities in gallons marked on them.

8. Ale or beer measure, formerly used in measuring beer, ale, and milk, is almost entirely discarded.

II. Dry Measure.

151. Dry Measure is used in measuring articles not liquid; as grain, fruit, salt, roots, ashes, etc.

Table.

2 Pints (pt.)	make	1 Quart	qt.
8 Quarts	"	1 Peck	pk.
4 Pecks	"	1 Bushel	bu.

1. In England, 8 bu. of 70 lb. each are called a *quarter*, used in measuring grain. The weight of the English quarter is ¼ of a long ton.

2. The wine and dry measures of the same denominations are of different capacities.

Metric Measure of Capacity.

152. The **Liter** is the unit of capacity. It is a volume equal to a cube, each edge of which measures *one tenth of a meter*. It is very nearly one quart, liquid measure.

Table.

10 Deciliters,	*dl.* = 1	**Liter.**
10 Liters,	*L.* = 1	Dekaliter.
10 Dekaliters,	*Dl.* = 1	Hectoliter.
10 Hectoliters,	*Hl.* = 1	Kiloliter, or Stere.

The *Hectoliter* is the *unit* in measuring liquids, grain, fruit, and roots in large quantities.

1. What is the cost of 3 liters of vinegar, at 25 cents per liter?

2. What must be paid for 4 barrels of oil, at 9 cents per gallon?

3. When apples are worth $1.20 a bushel, how much must one pay for half a peck?

4. A dealer buys 3 pecks of berries, dry measure, at 8 cents per quart. He sells the berries, by liquid measure, at 10 cents per quart. How much does he gain?

MISCELLANEOUS MEASURES.

Time Measure.

153. Time is the measure of duration.

Table.

60	Seconds (sec.)	make 1	Minute	min.
60	Minutes	" 1	Hour	hr.
24	Hours	" 1	Day	da.
7	Days	" 1	Week	wk.
365	Days	" 1	Common Year . . .	yr.
366	Days	" 1	Leap Year	yr.
12	Calendar Months	" 1	Year	yr.
100	Years	" 1	Century	C.

154. The calendar year is divided as follows:

Season.	No. of Month.	Names of Months.	Abbreviations.	No. of Days.
Winter,	1	January,	Jan.	31
	2	February,	Feb.	28 or 29
Spring,	3	March,	Mar.	31
	4	April,	Apr.	30
	5	May,	May	31
Summer,	6	June,	Jun.	30
	7	July,	July	31
	8	August,	Aug.	31
Autumn,	9	September,	Sept.	30
	10	October,	Oct.	31
	11	November,	Nov.	30
Winter,	12	December,	Dec.	31

1. The exact length of a solar year is 365 da. 5 hr. 48 min. 46 sec.; but for convenience it is reckoned 11 min. 14 sec. more than this, or 365 da. 6 hr. = 365¼ da. This ¼ day, in four years, makes one day, which, every fourth, or leap year, is added to the shortest month, giving it 29 days. The leap years are exactly divisible by 4, as 1856, 1860, 1864.

The number of days in each calendar month may be easily remembered by committing to memory the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and one day more
We add to it one year in four."

2. In most business transactions 30 days are called 1 month.

3. The centuries are numbered from the commencement of the Christian era; the months from the commencement of the year; the days from the commencement of the month, and the hours from the commencement of the day (12 o'clock, midnight). Thus, May 23d, 1890, 9 o'clock A.M., is the 9th hour of the 23d day of the 5th month of the 90th year of the 19th century.

1. How many hours are there in 4 weeks?

2. How many leap years have there been in the nineteenth century before Jan. 1, 1893?

3. In four years how many days are there?

4. If a business man draws $900 every three months for living expenses, what is the average daily cost?

Circular Measure.

155. Circular Measure, or Circular Motion, is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitude, determining locations of places and vessels, and computing difference of time.

Each circle, great or small, is divisible into the same number of equal parts, as quarters, called *quadrants;* twelfths, called *signs;* 360ths, called *degrees*, etc. Consequently the parts of unequal circles, although having the same names, are of unequal lengths.

Table.

60 Seconds (″)	make	1 Minute	′.
60 Minutes	"	1 Degree	°.
30 Degrees	"	1 Sign	S.
12 Signs, or 360°	"	1 Circle	C.

1. Minutes of the earth's circumference are called *geographic* or nautical miles.

2. The denomination, *signs*, is confined exclusively to Astronomy.

3. A degree has no fixed linear extent. When applied to any circle, it is always $\frac{1}{360}$ part of the circumference. But, strictly speaking, it is not any part of a circle.

4. 90° make a *quadrant* or right-angle.

5. 60° make a *sextant* or $\frac{1}{6}$ of a circle.

1. How many quadrants are there in 180°?
2. A steamer sails 2° 30′ in a day. How long will it take to sail a distance of 10°?
3. How many minutes are there in 3 signs?
4. How many seconds are there in 5 degrees?

156. COUNTING.

12 Units or things	make 1	Dozen	doz.
12 Dozen	" 1	Gross	gro.
12 Gross	" 1	Great gross	G. gro.
20 Units	" 1	Score	sc.

157. PAPER.

24 Sheets	make 1	Quire	qre.
20 Quires	" 1	Ream	rm.
2 Reams	" 1	Bundle	bdl.
5 Bundles	" 1	Bale	B.

BOOKS.

158. The terms *folio*, *quarto*, *octavo*, *duodecimo*, etc., indicate the number of leaves into which a sheet of paper is folded.

When a sheet is folded into	The book is called	And 1 sheet of paper makes
2 Leaves	a Folio,	4 pp. (pages).
4 "	a Quarto or 4to,	8 "
8 "	an Octavo or 8vo,	16 "
12 "	a Duodecimo or 12mo,	24 "
16 "	a 16mo,	32 "
18 "	an 18mo,	36 "

159. COPYING.

75 Words make 1 Folio or sheet of Common Law.
90 Words make 1 Folio or sheet of Chancery.

1. How many dozen are there in 3 score?

2. What must be paid for 5 gross of bottles, at 50 cents per dozen?

3. How many sheets of paper are there in 5 reams?

4. How many sheets are required for a 12mo volume of 360 pages?

5. What must be paid for copying 3000 words (common law), at 25 cents per page?

160. An **Aliquot Part** of a number is such a part as will exactly divide that number; thus, 3, 5, $7\frac{1}{2}$ are aliquot parts of 15.

161. ALIQUOT PARTS OF ONE DOLLAR.

50 Cents = $\frac{1}{2}$ of 1 Dollar.	$12\frac{1}{2}$ Cents = $\frac{1}{8}$ of 1 Dollar.
$33\frac{1}{3}$ Cents = $\frac{1}{3}$ of 1 Dollar.	10 Cents = $\frac{1}{10}$ of 1 Dollar.
25 Cents = $\frac{1}{4}$ of 1 Dollar.	$8\frac{1}{3}$ Cents = $\frac{1}{12}$ of 1 Dollar.
20 Cents = $\frac{1}{5}$ of 1 Dollar.	$6\frac{1}{4}$ Cents = $\frac{1}{16}$ of 1 Dollar.
$16\frac{2}{3}$ Cents = $\frac{1}{6}$ of 1 Dollar.	5 Cents = $\frac{1}{20}$ of 1 Dollar.

162. ALIQUOT PARTS OF A MILE.

160 Rods = $\frac{1}{2}$ Mile.	1760 Feet = $\frac{1}{3}$ Mile.
80 Rods = $\frac{1}{4}$ Mile.	880 Feet = $\frac{1}{6}$ Mile.
40 Rods = $\frac{1}{8}$ Mile.	440 Feet = $\frac{1}{12}$ Mile.

163. ALIQUOT PARTS OF AN ACRE.

80 Square Rods = $\frac{1}{2}$ Acre.	32 Square Rods = $\frac{1}{5}$ Acre.
40 Square Rods = $\frac{1}{4}$ Acre.	20 Square Rods = $\frac{1}{8}$ Acre.

164. ALIQUOT PARTS OF A TON.

10 Hund. lb. = $\frac{1}{2}$ Ton.	2 Hund. 50 lb. = $\frac{1}{8}$ Ton.
5 Hund. lb. = $\frac{1}{4}$ Ton.	2 Hund. lb. = $\frac{1}{10}$ Ton.
4 Hund. lb. = $\frac{1}{5}$ Ton.	1 Hund. lb. = $\frac{1}{20}$ Ton.

165. ALIQUOT PARTS OF A POUND AVOIRDUPOIS.

8 Ounces = $\frac{1}{2}$ Pound.	2 Ounces = $\frac{1}{8}$ Pound.
4 Ounces = $\frac{1}{4}$ Pound.	1 Ounce = $\frac{1}{16}$ Pound.

166. ALIQUOT PARTS OF TIME.

Parts of One Year.	*Parts of One Month.*
6 Months = $\frac{1}{2}$ Year.	15 Days = $\frac{1}{2}$ Month.
4 Months = $\frac{1}{3}$ Year.	10 Days = $\frac{1}{3}$ Month.
3 Months = $\frac{1}{4}$ Year.	6 Days = $\frac{1}{5}$ Month.
2 Months = $\frac{1}{6}$ Year.	5 Days = $\frac{1}{6}$ Month.
$1\frac{1}{2}$ Months = $\frac{1}{8}$ Year.	3 Days = $\frac{1}{10}$ Month.
1 Month = $\frac{1}{12}$ Year.	2 Days = $\frac{1}{15}$ Month.

1. At $16\frac{2}{3}$ cents per yard, how many yards of cloth can be bought for $2?

SOLUTION. — One sixth of a dollar is $16\frac{2}{3}$ cents; 6 yards can be bought for $1, 12 yards for $2.

2. At $8\frac{1}{3}$ cents per pound, how many pounds of sugar can be bought for $5?

3. A horse trots 1760 feet in 1 minute. How long will it take him to trot a mile?

4. At $300 per acre, what will 40 sq. rods of land cost?

SOLUTION. — Forty sq. rods equal $\frac{1}{4}$ of an acre; $\frac{1}{4}$ of $300 is $75, the cost of 40 sq. rods.

5. At 64 cents a pound, what will 2 ounces of spice cost?

6. The interest on a given sum of money, for one month, is $20. How much is the interest for 6 days? For one year?

7. A man walks 40 rods in 5 minutes. How long will it take him to walk a mile?

REDUCTION.

167. Reduction is the process of changing the denomination of a number, without altering its value.

168. Reduction to Lower Denominations is the process of changing a number of one denomination to other denominations of *less unit value* and is performed by *multiplication.*

Thus, $1 = 10 dimes = 100 cents; 1 yd. = 3 ft. = 36 in.

EXAMPLES.

1. Reduce 6 gal. 2 qt. 1 pt. to pints.

OPERATION.

6 gal. 2 qt. 1 pt.
4
26 qt.
2
Ans. 53 pt.

SOLUTION. — Since in 1 gal. there are 4 qt., in 6 gal. there are 6 × 4 qt. = 24 qt., and the 2 qt. in the given number added, make 26 qt. in 6 gal. 2 qt. Since in 1 qt. there are 2 pt., in 26 qt. there are 26 × 2 pt. = 52 pt., and the 1 pt. in the given number added, make 53 pt. in the given compound number. As either factor may be used as a multiplicand (**61**), we may consider the numbers in the descending scale as multipliers.

Rule. I. *Multiply the number in the highest denomination of the given number by that number of the scale which will reduce it to the next lower denomination, and add to the product the given number, if any, of that lower denomination.*

II. *Proceed in the same manner with the result obtained in each lower denomination, until the reduction is brought to the denomination required.*

2. How many ounces are there in 8 lb. 10 oz.?

Ans. 138 oz.

3. How many pence are there in £12 6*s.* 9*d.*?

4. How many inches are there in 4 yd. 1 ft. 10 in. ?

5. How many rods are there in 3 mi. 226 rd. ?

6. How many farthings are there in 18 *s.* 8 *d.* 3 far. ?

7. Reduce 3 lb. 9 oz. 12 pwt. to pennyweights.

8. Change 2 hhd. 15 gal. 2 qt. to pints.

9. Reduce 4 da. 5 hr. to minutes. *Ans.* 6060 min.

10. Reduce 10 bu. 1 pk. 6 qt. to pints. *Ans.* 668 pt.

11. Reduce 14 A. 140 sq. rd. to square rods.

12. Reduce 4 cd. 3 cd. ft. 9 cu. ft. to cubic inches.

13. Reduce 4 yr. 7 mo. to hours. *Ans.* 39600 hr.

14. Change 2 T. 11 cwt. to pounds. *Ans.* 5100 lb.

15. Change 9 lb. 9 oz. 10 pwt. to grains.

16. Change 5 lb. 3 oz. 6 pwt. to grains.

17. Change 3 mi. 240 rd. to feet. *Ans.* 19800 ft.

18. Reduce 40 chains to links. *Ans.* 4000 l.

19. Reduce 28 sq. rd. 12 sq. yd. 4 sq. ft. to square inches. *Ans.* 1113840 sq. in.

20. In 16 A. 4 sq. ch. 8 P. 80 sq. l. how many square links are there ? *Ans.* 1645080 sq. l.

21. Change 12 tons of round timber to cubic inches.

22. Reduce 8 bbl. 26 gal. to pints. *Ans.* 2224 pt.

23. Reduce 4 pipes to quarts. *Ans.* 2016 qt.

24. Reduce 23 bu. 3 pk. to pints. *Ans.* 1520 pt.

25. Reduce 8 S. 18° 40′ to minutes. *Ans.* 15520′.

26. Reduce 15° to seconds. *Ans.* 54000″.

27. Reduce 2 months to minutes. *Ans.* 86400 min.

28. Change 2 reams 10 quires to sheets.

29. Change 40 score to single things. *Ans.* 800.

30. Change 14 great gross to dozens.

31. How many seconds are there in 30° 20′ 24″ ?

32. How many hours are there in the 3 autumn months ?

33. How many minutes are there in the 3 summer months ?

34. Change 75 cords to cubic feet.

169. Reduction to Higher Denominations is the process of changing a number of one denomination to others of *greater unit value*, and is performed by *division*.

Thus, 100 cents = $1.

EXAMPLES.

1. Reduce 53 pints to gallons.

OPERATION.

2 | 53
4 | 26 qt. + 1 pt.
6 gal. + 2 qt.

Ans. 6 gal. 2 qt. 1 pt.

SOLUTION. — Dividing the given number of pints by 2, because there are ½ as many quarts as pints, we obtain 26 quarts plus a remainder of 1 pt. Next we divide 26 quarts by 4, because there are ¼ as many gallons as quarts, and we obtain 6 gallons and a remainder of 2 qt. This last quotient, with the several remainders annexed, forms the answer.

2. Reduce 4902 inches to rods.

OPERATION.

12 | 4902
16½ | 408 ft. + 6
2
33 | 816
24 rd. + 24/2 = 12 ft.

Ans. 24 rd. 12 ft. 6 in.

SOLUTION. — We divide successively by the numbers in the ascending scale in the same manner as in the preceding example. But in dividing the 408 ft. by 16½, we first reduce 408 ft. to *halves* by multiplying by 2, and we have 816 *halves*; and reducing 16½ to *halves*, we have 33 *halves*. Then dividing 816 by 33 we obtain 24 rd. plus a remainder of 24 *halves* = to 12 ft., which, with the preceding remainder annexed to the last quotient, gives the answer, 24 rd. 12 ft. 6 in.

Rule. I. *Divide the given number by that number of the scale which will reduce it to the next higher denomination.*

II. *Divide the quotient by the next higher number in the scale; and so proceed to the highest denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.*

3. How many pounds are there in 3460 ounces?
Ans. 216 lb. 4 oz.
4. How many shillings are there in 556 farthings?
Ans. 11 *s.* 7 *d.*
5. How many yards are there in 1242 inches?
6. How many gallons are there in 2347 pints?
7. Reduce 23547 troy grains to pounds.
Ans. 4 lb. 1 oz. 1 pwt. 3 gr.
8. Reduce 1597 quarts to bushels.
Ans. 49 bu. 3 pk. 5 qt.
9. Reduce 107520 oz. avoirdupois to pounds.
10. How many hours are there in 28635 sec.?
Ans. 7 hr. 57 min. 15 sec.
11. Change 10000″ to degrees. *Ans.* 2° 46′ 40″.
12. Change 11521 gr. troy weight to pounds.
13. Change 3561829 seconds to weeks.
14. Reduce 67893 cu. ft. to cords.
15. Change 1491 pounds to hundred weight.
16. Change 12244 pints to hogsheads.
17. Reduce 25600 sq. rd. to acres. *Ans.* 160 A.
18. How many miles are there in 51200 rd.?
Ans. 160 mi.
19. How many barrels are there in 6048 gills?
Ans. 6 bbl.
20. Change 316800 inches to miles. *Ans.* 5 mi.
21. Change 1728 to gross. *Ans.* 12 gross.
22. Change 4060 to score. *Ans.* 203 score.
23. Reduce 1435 feet to fathoms.
24. Reduce 10000 sheets of paper to reams.
Ans. 20 reams 16 quires 16 sheets.
25. Reduce 27878400 sq. ft. to square miles.
26. Reduce 7600 inches to rods.
27. Reduce 10000 feet to inches.
28. Reduce 8000 pints to pecks.

PROMISCUOUS EXAMPLES.

1. Reduce 4 dollars 67 cents to cents. *Ans.* 467 cents.
2. Reduce 3724 cents to dollars. *Ans.* $37.24.
3. Reduce 9690 cents to dollars. *Ans.* $96.90.
4. Reduce 8 dollars to cents. *Ans.* 800 cents.
5. How many pounds are there in 91751 farthings? *Ans.* £95 11*s.* 5*d.* 3 far.
6. How many grains are there in 3 lb. 4 oz. 7 pwt.?
7. How many pounds are there in 3 tons of cheese?
8. How much will 4 cheeses cost, each weighing 36 pounds, at 9 cents a pound? *Ans.* $12.96.
9. How much would 2 lb. 8 oz. 12 pwt. of gold dust be worth, at 72 cents a pwt.? *Ans.* $469.44.
10. Bought 1 T. 15 cwt. 36 lb. of sugar at 7 cents a pound. What did it cost? *Ans.* $247.52.
11. Paid $25.50 for one hogshead of molasses, and sold it all at $.50 a gallon. What was the whole gain?
12. How many pounds in 6 barrels of flour?
13. How many bushels of oats (32 lb.) in a load weighing 1280 pounds? *Ans.* 40 bu.
14. How many bushels (60 lb.) of wheat in a load weighing 2175 pounds? *Ans.* 36 bu. 15 lb.
15. A grocer bought 3 barrels of flour at $6 a barrel, and sold it all at 4 cents a pound. What did he gain on the whole? *Ans.* $5.52.
16. In a board 12 feet long and 2 feet wide, how many square feet are there? *Ans.* 24 sq. ft.
17. In a block of marble 6 feet long and 3 feet square, how many cubic feet are there? *Ans.* 54 cu. ft.
18. How many cubic feet are there in a pile of wood 26 feet long, 6 feet high, and 3 feet wide? How many cords? *Ans.* 468 cu. ft.; or 3 Cd. 84 cu. ft.

19. In 259200 cubic inches of hewn timber how many tons are there? *Ans.* 3 T.

20. How many square rods are there in a field 90 rods long and 75 rods wide? How many acres?

Ans. 42 A. 30 sq. rd.

21. A pond of water measures 3 fathoms 2 feet 9 inches in depth. How many inches deep is it?

Ans. 249 in.

22. What will 3 miles of telegraph cable cost at 12 cents a foot? *Ans.* $1900.80.

23. What is the age in years of a man who is 3 score and 5 years old? *Ans.* 65 years.

24. How much will I receive for a load of wheat weighing 2760 pounds at $1.50 per bushel? *Ans.* $69.

25. How many cubic feet are there in a stick of timber 32 feet long, 2 feet wide, and 1 foot thick? *Ans.* 64 cu. ft.

26. How many square feet are there in 1 acre?

27. In 176 yards how many rods are there?

28. A pile of wood is 16 feet long, 8 feet high, and 8 feet wide. How much is it worth at $3.50 a cord?

Ans. $28.

29. What would be the value of a city lot 40 feet wide and 120 feet long, at $2 a square foot? *Ans.* $9600.

30. A grocer bought 4 barrels of cider, at $2 a barrel, and after converting it into vinegar, he retailed it at $.15 a gallon. What was his whole gain? *Ans.* $10.90.

31. At $.06 a pint, how much molasses can be bought for $4.26? *Ans.* 8 gal. 3 qt. 1 pt.

32. What will be the cost of a hogshead of sperm oil at $.08 a gill? *Ans.* $161.28.

33. An innkeeper bought a load of 40 bushels of oats, at $.36 a bushel, and retailed it at $.25 a peck. What did he make on the load? *Ans.* $25.60.

34. Change 120 gross to score. *Ans.* 864 score.

35. If a man walks 4 miles an hour, and 10 hours a day, how many miles can he walk in 24 days? *Ans.* 960 mi.

36. What will be the cost of 2 bu. 1 pk. 6 qt. of timothy seed, at 10 cents a quart? *Ans.* $7.80.

37. What would be the value of a silver goblet, weighing 8 oz. 14 pwt., at $.15 a pwt.? *Ans.* $26.10.

38. What will 16 reams of paper cost, at $.20 a quire? *Ans.* $64.

39. If 1 bushel of wheat makes 45 pounds of flour, how many pounds will 500 bushels make? How many barrels? *Ans.* 114 bbl. 156 lb.

40. Bought a gold chain weighing 2 oz. 18 pwt., at $.90 a pwt. What did it cost? *Ans.* $52.20.

41. How many minutes more are there in the summer than in the autumn months? *Ans.* 1440 min.

42. How much will it cost to dig a cellar 24 ft. long, 18 ft. wide, and 6 feet deep, at 1 cent a cubic foot?

Ans. $25.92.

43. How many boxes, each containing 12 pounds, can be filled from a hogshead of sugar containing 9 cwt.?

Ans. 75 boxes.

44. What will be the cost of 5 bales of cloth, each bale containing 15 pieces, and each piece measuring 26 yards, at $1.75 a yard?

45. If a cannon ball goes at the rate of 10 miles a minute, how many miles would it go at the same rate in 2 hours? *Ans.* 1200 miles.

46. At $.11 a pound, what will be the cost of 3 cwt. 71 lb. of sugar? *Ans.* $40.81.

47. What will be the cost of a load of oats weighing 1456 lb., at $.37½ per bu.? *Ans.* $17.0625.

48. If one bushel of wheat will make 45 lb. of flour, how many barrels will 1000 bu. make?

Ans. 229 bbl. 116 lb.

ADDITION.

170. Compound numbers are added, subtracted, multiplied, and divided in the same general way as simple numbers. The only modification of the operations and rules is that required for borrowing, carrying, and reducing by a *varying*, instead of a *uniform scale.*

EXAMPLES.

1. What is the sum of 10 £ 7 *s.* 3 *d.*, 17 £ 5 *s.* 4 *d.*, and 2 £ 9 *s.* 6 *d.*?

OPERATION.

£.	*s.*	*d.*
10	7	3
17	5	4
2	9	6
30	2	1

SOLUTION. — Arranging the numbers in columns, placing units of the same denomination in the same column, we first add the column of the lowest denomination. The sum is 13 *d.*, equal to 1 *s.* 1 *d.* We write the 1 *d.* under the column of pence, and add the 1 *s.* to the column of shillings; the sum of the shillings is 22 *s.*, equal to 1 £ 2 *s.* We write the 2 *s.* under the column of shillings. Adding the 1 £ to the column of pounds, the sum is 30 £. The entire sum is 30 £ 2 *s.* 1 *d.*

Rule. I. *Write the numbers so that those of the same unit value stand in the same column.*

II. *Beginning at the right hand, add each denomination as in simple numbers, carrying to each succeeding denomination 1 for as many units as it takes of the denomination added, to make one of the next higher denomination.*

Add:

2.

£	*s.*	*d.*	far.
47	10	9	1
25	6	4	3
36	18	0	2
Ans. 109	15	2	2

3.

£	*s.*	*d.*	far.
10	10	4	1
	9	5	2
14	4	0	0

4.

£	s.	d.	far.
24	2	3	1
102	4	2	0
38	9	0	1
42	5	1	0

5.

£	s.	d.	far.
3	12	5	1
	16	0	2
5	9	6	0
	18	7	1

6.

£	s.	d.	far.
27	14	4	3
106	10	1	2
16	12	5	0
52	16	9	1
16	18	10	3

7.

£	s.	d.	far.
16	11	8	2
26	9	5	1
11	10	0	3
4	6	2	0
34	14	9	2

8.

£	s.	d.	far.
2	5	4	1
1	3	1	2
4	16	5	0
910	8	2	2
88	9	4	8

9.

£	s.	d.	far.
12	2	5	1
69	15	6	2
715	10	7	1
320	16	3	3
18	9	4	17

171. In actual business transactions, it is very seldom that more than one denomination is mentioned. Two bushels and 2 pecks are called two bushels and a half; 3 pounds and 4 ounces, $3\frac{1}{4}$ pounds; 1 quart and 1 pint, $1\frac{1}{2}$ quart, or, more often, 3 pints; 1 mile and 2640 feet, $1\frac{1}{2}$ miles, etc.

The ordinary business practice respecting compound numbers is to express all lower denominations as common or decimal fractions of the highest. Computations are then made as in other cases where fractions occur.

172. 1. What is the sum of 2 A. 80 sq. rd., 3 A. 32 sq. rd., and 5 A. 40 sq. rd.?

OPERATION.

A.	A.
$2\frac{1}{2}$	2.5
$3\frac{1}{5}$	3.2
$5\frac{1}{4}$	5.25
$10\frac{19}{20}$	10.95

SOLUTION. — Expressing the lower denominations as fractions (**163**), common or decimal, the result is obtained by the ordinary forms of addition more quickly and easily than when the varying scale is used.

2. What is the sum of 3 mo. 10 da., 2 mo. 6 da., 7 mo. 15 da.?

3. What is the sum of 4 mi. 40 rd., 7 mi. 1760 ft., 3 mi. 80 rd., 2 mi. 880 ft.?

4. What is the sum of 3 T. 250 lb., 2 T. 5 cwt., 4 T. 4 cwt.?

5. What is the amount of $\$1.33\frac{1}{3}$, $\$4.25$, $\$7.12\frac{1}{2}$, $\$9.16\frac{2}{3}$, $\$12.50$, $\$3.08\frac{1}{3}$, and $\$3.75$?

6. A grocer bought 4 hhd. of sugar; the first weighed 11 cwt. 71 lb.; the second 10 cwt. 41 lb.; the third 10 cwt. 22 lb.; and the fourth 9 cwt. 75 lb. How much did the whole weigh? *Ans.* 2 T. 2 cwt. 9 lb.

7. A man has a farm divided into three fields; the first contains 26 A. 32 sq. rd.; the second 48 A. 120 sq. rd.; and the third 35 A. 80 sq. rd. How many acres are there in the farm?

8. If a printer one day uses 2 bundles 1 ream 10 quires of paper, the next day 3 bundles 1 ream 12 quires 20 sheets, and the next, 4 bundles 9 quires, how much does he use in the three days?

Ans. 10 bundles 1 ream 11 quires 20 sheets.

9. A tailor used, in one year, 3 gross 6 doz. 10 buttons; another year, 2 gross 9 doz. 9 buttons, and another year, 4 gross 7 doz. buttons; how many did he use in the three years?

SUBTRACTION.

EXAMPLES.

173. 1. From 24 yr. 6 mo. 5 da. 12 hr. take 14 yr. 9 mo. 10 da. 7 hr.

OPERATION.

	yr.	mo.	da.	hr.
	24	6	5	12
	14	9	10	7
Ans.	9	8	25	5

SOLUTION. — Writing the subtrahend under the minuend, placing units of the same denomination under each other, we subtract 7 hr. from 12 hr. and write the remainder, 5 hr., underneath. Since we cannot subtract 10 da. from 5 da., we add 1 mo. or 30 da. to the 5 da., and subtract 10 da. from the sum, 35 da., and write the remainder, 15 da., underneath. Having added 30 da. or 1 mo. to the minuend, we now add 1 mo. to the 9 mo. in the subtrahend, making 10 mo.; but as we cannot take 10 mo. from 6 mo. we add 1 yr. or 12 mo. to the 6 mo., making 18 mo., and subtracting 10 mo. from 18 mo. we write the remainder, 8 mo., under the denomination of months. Having added 1 yr. to the minuend, we now add 1 yr. to the 14 yr. in the subtrahend, and subtracting 15 yr. from 24 yr. as in simple numbers, we write the remainder, 9 yr., under the denomination of years.

Rule. I. *Write the subtrahend under the minuend, so that units of the same denomination stand under each other.*

II. *Beginning at the right hand, subtract each denomination separately, as in simple numbers.*

III. *If the number of any denomination in the subtrahend exceeds that of the same denomination in the minuend, add to the number in the minuend as many units as make one of the next higher denomination, and then subtract; in this case add 1 to the next higher denomination of the subtrahend before subtracting. Proceed in the same manner with each denomination.*

174. Business men use the fractional forms (see **171**) in subtraction, in preference to the varying scale.

2. From 6 lb. 4 oz. take 4 lb. 8 oz.

OPERATION.

lb.	lb.
$6\frac{1}{4}$	6.25
$4\frac{1}{2}$	4.50
$1\frac{3}{4}$ lb.	1.75 lb.

SOLUTION. — Expressing the lower denomination in fractional form (**165**), the result is obtained as in ordinary subtraction of fractions.

3.

	cwt.	qr.	lb.	oz.
From	18	1	14	9
Take	5	2	10	6
Rem.	12	2	4	3

4.

	gal.	qt.	pt.
	48	2	1
	42	3	0
	5	3	1

5.

yr.	mo.	da.	hr.	min.
12	7	23	15	11
8	8	4	2	10

6.

bu.	pk.
104	2
56	3

7.

mi.	rd.	yd.	ft.	in.
40	130	3	2	10
14	115	2	1	1

8.

A.	P.
400	125
325	130

9. From 125 mi. 240 rd. take 90 mi. 185 rd.

Ans. 35. mi. 55 rd.

10. A man bought 60 gal. of molasses, and sold 42 gal. 3 qt. How much remained? *Ans.* 17 gal. 1 qt.

11. Mr. Owen bought 9 T. 14 cwt. of coal, and having burned 4 T. 15 cwt. sold the remainder. How much did he sell? *Ans.* 4 T. 19 cwt.

12. If from a tub of butter containing 1 cwt. 21 lb. there are sold 24 lb. 8 oz. how much remains?

Ans. 96 lb. 8 oz.

13. From a pile of wood containing 42 Cd. 5 cd. ft. there were sold 16 Cd. 6 cd. ft. How much remained?

Ans. 25 Cd. 7 cd. ft.

MULTIPLICATION.

EXAMPLES.

175. 1. A farmer has 8 fields, each containing 4 A. 107 P. How much land has he in all?

OPERATION.

A.	P.
4	107
	8
37	56

SOLUTION. — In 8 fields there is 8 times as much land as in 1 field. We write the multiplier under the lowest denomination of the multiplicand, and proceed thus: 8 times 107 P. are 856 P., equal to 5 A. 56 P. We write the 56 P. under the number multiplied. Then, 8 times 4 A. are 32 A., and 5 A. added make 37 A., which are written under the same denomination in the multiplicand, and the work is done.

Rule. I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply as in simple numbers, and carry as in addition of compound numbers.*

176. When more convenient, reduce to the fractional forms, as previously illustrated, and perform the operations as in ordinary examples in multiplication of fractions.

2.

	gal.	qt.	pt.
	20	2	1
			3
Ans.	61	3	1

3.

	bu.	pk.	qt.	pt.
	9	2	6	1
				4
	38	3	2	0

4.

lb.	oz.	pwt.	gr.
12	8	14	16
			5
63	7	13	8

5.

T.	cwt.	lb.	oz.
10	15	20	8
			6
64	11	23	0

6. Multiply 14 A. 106 P. by 8. *Ans.* 117 A. 48 P.

7. Multiply 6 yd. 2 ft. by 12.

8. Multiply 1 yr. 1 mo. 15 da. by 24. (See **166.**)

9. Multiply 24 bu. 2 pk. by 10.

10. What will 6 yd. of cloth cost at $\$ 3.33\frac{1}{3}$ per yard?

11. Multiply £84 12 *s.* 6 *d.* 2 far. by 9.

12. If a pipe discharges 4 hhd. 20 gal. 3 qt. of water in 1 hour, how much will it discharge in 5 hours, at the same rate? *Ans.* 21 hhd. 40 gal. 3 qt.

13. If a load of coal weighs 1 T. 4 cwt., what will be the weight of 6 loads?

14. If 1 acre of land produces 26 bu. 3 pk. of wheat, how much will 11 acres produce?

15. If a man travels 30 mi. 80 rd. in one day, how far will he travel in 9 days, at the same rate?

16. Multiply 1 mi. 1760 ft. by 12.

17. If a yard of cloth costs £ 2 10 *s.* 9 *d.*, what will 18 yards cost? *Ans.* £ 45 13 *s.* 6 *d.*

18. If a person averages 8 hr. 20 min. of sleep daily, how much will he sleep in 30 days? *Ans.* 10 da. 10 hr.

19. How many cords of wood in 8 piles, each containing 40 cd. ft.?

20. If each silver table-spoon weighs 1 oz. 12 pwt. 16 gr., what is the weight of 1 dozen spoons?

21. If the moon's average daily motion is 33° 10′ 35″, how much of her orbit does she traverse in 21 days?

22. How much land is there in 12 lots, each containing 2 A. 120 P.? *Ans.* 33 A.

23. How many bushels of wheat are there in 48 sacks, each containing 165 pounds? *Ans.* 132 bu.

24. If a locomotive moves 196 rd. in one minute, how far will it move in one hour? *Ans.* 36 mi. 240 rd.

25. If a family consumes 2 gal. 1 qt. of molasses in 1 week, how much will it consume in 1 year?

26. How many bushels of corn will a field of 9 acres produce, if it produces 25 bu. 2 pk. 3 qt. to the acre?

DIVISION.

EXAMPLES.

177. 1. If 4 acres of land produce 102 bu. 2 pk. 2 qt. of wheat, how much will 1 acre produce?

OPERATION.

	bu.	pk.	qt.	pt.
4)	102	3	2	
	25	2	6	1

SOLUTION. — One acre will produce $\frac{1}{4}$ as much as 4 acres. Writing the divisor on the left of the dividend, we divide 102 bu. by 4, and obtain a quotient of 25 bu., and a remainder of 2 bu. We write the 25 bu. under the denomination of bushels, and reduce the 2 bu. to pecks, making 8 pk., and the 3 pk. of the dividend added make 11 pk. Dividing 11 pk. by 4, we obtain a quotient of 2 pk., and a remainder of 3 pk.; writing the 2 pk. under the pecks, we next reduce the 3 pk. to quarts, adding the 2 qt. of the dividend, making 26 qt., which divided by 4 give a quotient of 6 qt., and a remainder of 2 qt. Writing the 6 qt. under the quarts, and reducing the remainder, 2 qt., to pints, we have 4 pt., which divided by 4 give a quotient of 1 pt., which we write under the pints, and the work is done.

OPERATION.

bu.	pk.
46)132	1(2 bu.
92	
40	
4	
161	(3 pk.
138	
23	
8	
184	(4 qt.
184	

Ans. 2 bu. 3 pk. 4 qt.

2. A farmer put 132 bu. 1 pk. of apples into 46 barrels. How many bu. did he put into 1 barrel?

SOLUTION. — When the divisor is large, we divide by long division, as shown in the operation.

From these examples we derive the following rule:

Rule. I. *Divide the highest denomination as in simple numbers, and each successive denomination in the same manner, if there is no remainder.*

II. *If there is a remainder after dividing any denomination, reduce it to the next lower denomination, adding in the given number of that denomination, if any, and divide as before.*

III. *The several partial quotients will be the quotient required.*

178. In division of compound numbers, it is frequently desirable to employ the fractional or decimal forms instead of the varying scale. This is illustrated by the fourth and fifth examples.

3.

	A.	P.
2)	95	80
	47	120

4.

	A.
2)	$95\frac{1}{2}$
	$47\frac{3}{4}$

5.

	A.
2)	95.5
	47.75

6.

	bu.	pk.	qt.
6)	88	3	4
	14	3	2

7.

	wk.	da.	hr.
7)	30	3	12
	4	2	12

8.

	lb.	oz.	pwt.	gr.
3)	52	4	16	18
	17	5	12	6

9.

	mi.	rd.
5)	12	160
	2	160

10.

	gal.	qt.	pt.
9)	376	3	1
	41	3	1

11. Divide £185 17 *s.* 6 *d.* by 8.

Ans. £23 4 *s.* 8 *d.* 1 far.

12. Divide 16 lb. 14 oz. by 6. *Ans.* 2 lb. 13 oz.

13. Divide 360 A. 80 sq. rd. by 7.

14. Divide 193 bu. 2 pk. by 9.

15. Divide 9 hhd. 28 gal. 2 qt. by 12.

Ans. 49 gal. 2 qt. 1 pt.

16. Divide 328 yd. 1 ft. 6 in. by 3.

17. Divide 16 cwt. by 32.

18. If a vessel makes 3 trips in 4 months and 15 days, how much time must be allowed for each trip?

PROMISCUOUS EXAMPLES.

1. A farmer raised 200 bu. 2 pk. of barley, 175 bu. 3 pk. of corn, 320 bu. 1 pk. of oats, and 225 bu. 2 pk. of rye. What was the whole quantity of grain raised?

2. A person having bought 325 A. 80 P. of land, sold 150 A. 65 P. of it. How much had he remaining?

3. What is the whole weight of 72 hhd. of sugar, each weighing 12 cwt. 75 lb.? *Ans.* 45 T. 18 cwt.

4. If a railroad car runs 148 mi. 160 rd. in 8 hr., what is the average rate of speed per hour?

5. A grocer having purchased 98 cwt. 50 lb. of sugar, sold 10 cwt. 45 lb. to one man, and 18 cwt. 16 lb. to another. How much remained unsold?

6. Bought 12 tea-spoons, each weighing 16 pwt. 20 gr., and 6 table-spoons, each weighing 1 oz. 12 pwt. What was their total weight? *Ans.* 1 lb. 7 oz. 14 pwt.

7. A farmer raised 24 T. 17 cwt. of hay; he sold 5 loads, each weighing 1 T. 8 cwt. 21 lb. How much has he remaining? *Ans.* 17 T. 15 cwt. 95 lb.

8. A jeweler having 36 lb. 10 oz. 14 pwt. of silver, uses 21 lb. 6 oz. of it, and then manufactures the remainder into 8 teapots. What is the weight of each?

Ans. 1 lb. 11 oz. 1 pwt. 18 gr.

9. A man purchasing 2 A. 140 P. of land, reserves $\frac{1}{2}$ an acre for his own use, and divides the remainder into 4 equal lots. How much does each lot contain? *Ans.* 95 P.

10. How many pounds of sugar in 28 barrels, each containing 3 cwt. 42 lb.? *Ans.* 9576 lb.

11. If from a piece of land containing 5 A. 120 P., 2 A. 72 P. are taken away, how many square rods will remain?

12. Divide a tract of land containing 1299500 square rods into 25 farms of equal area. How much will there be in each? *Ans.* 324 A. 140 P.

13. A merchant buys 3 hogsheads of molasses at $.30 a gallon, and sells it at $.45. What does he gain on the whole? *Ans.* $28.35.

14. What is the cost of 3 chests of tea, each weighing 2 cwt. 68 lb., at $.84 a pound? *Ans.* $675.36.

15. How many steps, of 30 inches each, must a person take in walking 12 miles?

16. If a man buys 10 bushels of chestnuts, at $3 a bushel, and sells them at 10 cents a pint, what is his whole gain? *Ans.* $34.

17. How many times will a wheel 13 ft. 4 inches in circumference turn around in going 12 miles?

Ans. 4752.

18. If 8 horses eat 12 bu. 3 pk. of oats in 3 days, how many bushels will 20 horses eat in the same time?

Ans. 31 bu. 3 pk. 4 qt.

19. How much sugar, at 9 cents a pound, must be given for 2 cwt. 43 lb. of pork at 6 cents a pound?

Ans. 162 lb.

20. How many cubic feet in a room 18 feet long, 16 feet wide, and 10 feet high?

21. A person wishes to ship 720 bushels of potatoes in barrels, which shall hold 3 bu. 3 pk. each. How many barrels must he use? *Ans.* 192 bbl.

22. How many rods of fence will inclose a farm 1 mile square? *Ans.* 1280 rd.

23. If granite weighs 175 pounds a cubic foot, what is the weight of a cubic yard? *Ans.* 2 T. 7 cwt. 25 lb.

PERCENTAGE.

179. Per Cent is a term derived from the Latin words *per centum*, and signifies *by the hundred*, or *hundredths*, that is, a certain number of parts of each *one hundred* parts, of whatever denomination.

Thus, by 5 per cent is meant 5 cents out of every 100 cents, $5 out of every $100, 5 bushels out of every 100 bushels, etc. Therefore, 5 per cent equals 5 hundredths = .05 = $\frac{5}{100} = \frac{1}{20}$. 8 per cent equals 8 hundredths = .08 = $\frac{8}{100} = \frac{2}{25}$.

The sign used to denote per cent is %; 6% is read 6 per cent.

180. Percentage is such a part of a number as is indicated by the per cent.

181. The **Base** of percentage is the number on which the percentage is computed.

182. Since per cent is any number of hundredths, it is usually expressed in the form of a *decimal* or a *common fraction*, as in the following table:

TABLE.

	Decimals.	Common Fractions.	Lowest Terms.		Decimals.	Common Fractions.	Lowest Terms.
1% =	.01 =	$\frac{1}{100}$ =	$\frac{1}{100}$	10% =	.10 =	$\frac{10}{100}$ =	$\frac{1}{10}$
2% =	.02 =	$\frac{2}{100}$ =	$\frac{1}{50}$	15% =	.15 =	$\frac{15}{100}$ =	$\frac{3}{20}$
4% =	.04 =	$\frac{4}{100}$ =	$\frac{1}{25}$	16% =	.16 =	$\frac{16}{100}$ =	$\frac{4}{25}$
5% =	.05 =	$\frac{5}{100}$ =	$\frac{1}{20}$	20% =	.20 =	$\frac{20}{100}$ =	$\frac{1}{5}$
6% =	.06 =	$\frac{6}{100}$ =	$\frac{3}{50}$	25% =	.25 =	$\frac{25}{100}$ =	$\frac{1}{4}$
7% =	.07 =	$\frac{7}{100}$ =	$\frac{7}{100}$	50% =	.50 =	$\frac{50}{100}$ =	$\frac{1}{2}$
8% =	.08 =	$\frac{8}{100}$ =	$\frac{2}{25}$	100% =	1.00 =	$\frac{100}{100}$ =	1

EXAMPLES.

183. To find the percentage of any number.

1. A man having $120, paid out 5% of it for groceries; how much did he pay out?

OPERATION.

$120
.05
$6.00

SOLUTION. — Five hundredths of $120 is $6; he paid $6.

Rule. *Multiply the given number or quantity by the rate per cent, expressed decimally, and point off as in decimals.*

2. What is 4% of $300? *Ans.* $12.
3. What is 3% of $175? *Ans.* $5.25.
4. What is 5% of 450 pounds?
5. What is 6% of 65 gallons? *Ans.* 3.9 gal.
6. What is 9% of 200 sheep? *Ans.* 18 sheep.
7. What is 7% of $97? *Ans.* $6.79.
8. What is 10% of $12.50? *Ans.* $1.25.
9. What is 40% of 840 men? *Ans.* 336 men.
10. What is 25% of 740 miles?
11. A man having $4000, invests 25% of it in land. What sum does he invest? *Ans.* $1000.
12. A man bought 1500 barrels of apples, and found on opening them that 12% of them were spoiled. How many barrels did he lose? *Ans.* 180 bbl.
13. A farmer having 180 sheep, sold 45% of them and kept the remainder. How many did he sell and how many did he keep? *Ans.* He kept 99.
14. Having deposited $1275 in a bank, I draw out 8% of it. How much remains? *Ans.* $1173.

COMMISSION.

184. An **Agent,** or **Broker** is a person who transacts business for another.

185. A **Commission Merchant** is an agent who buys and sells goods for another.

186. Commission is the fee or compensation of an agent, or commission merchant.

EXAMPLES.

187. To find the commission or brokerage on any sum of money.

1. A commission merchant sells butter and cheese to the amount of $1540. What is his commission at 5%?

OPERATION.
$1540 × .05 = $77, *Ans.*

SOLUTION. — Five per cent of $1540 is $77; his commission is $77.

Rule. *Multiply the given sum by the rate per cent, expressed decimally; the result will be the commission or brokerage.*

2. What commission must be paid for collecting $3840, at 3%? *Ans.* $115.20.

3. A commission merchant sells goods to the amount of $5487.50. What is his commission at 2%. *Ans.* $109.75.

4. An agent buys 5460 bushels of wheat at $1.50 a bushel. How much is his commission for buying, at 4%? *Ans.* $327.60.

5. A commission merchant sells 400 barrels of potatoes at $2.25 a barrel, and 345 barrels of apples at $3.20 a barrel. How much is his commission for selling, at 5%?

PROFIT AND LOSS.

188. Profit and **Loss** are commercial terms, used to express the gain or loss in business transactions, which is usually reckoned at a certain per cent on the prime or first cost of articles.

EXAMPLES.

189. To find the amount of profit or loss, when the cost and the gain or loss per cent are given.

1. A man bought a horse for $135, and afterwards sold it for 20% more than he gave. How much did he gain?

OPERATION.
$135 × .20 = $27, *Ans.*

SOLUTION. — Twenty per cent of $135, or $27, is the sum gained.

Rule. *Multiply the cost by the rate per cent, expressed decimally.*

2. I bought a horse for $150, and sold it at 15% profit. What was my gain? *Ans.* $22.50.

3. I bought 25 cords of wood at $3.50 a cord, and sold it so as to gain 33%. What did I make? *Ans.* $28.87½.

4. I paid 7 cents a pound for 2480 pounds of pork, and afterwards lost 10% on the cost, in selling it. What was my whole loss? *Ans.* $17.36.

5. I bought 1000 bushels of wheat at $1.25 a bushel, and sold the flour at 18% advance on the cost of the wheat. What was my whole gain? *Ans.* $225.

6. A grocer bought 6 barrels of sugar, each containing 220 pounds, at 7½ cents a pound, and sold it at 20% profit. What was the whole gain? *Ans.* $19.80.

7. At what price must an article that cost $10 be sold to gain 20%? 25%? 40%? 50%? 100%? 150%? 200%?

8. Some cloth bought at $4 a yard, was sold at 25% advance. What was the gain?

9. I bought a cow for $48, and sold it at a loss of 12½%? What was my loss?

10. A carriage that cost $120 was sold at 10% loss. What was the loss?

11. I bought a horse for $175, and sold it at 20% profit. What were the gain and the selling price?

12. An auctioneer sold goods to the amount of $3640. What was his commission at 2½%?

13. A grocer bought 1280 lb. of sugar, at $.07 a pound, and sold it at a profit of 15%. What was his gain?

14. A farm that cost $5840, was sold at a loss of 18%. What was it sold for?

15. What commission must be paid for collecting $2650, at 3¼%? At 4½%? At 5%?

16. A bought 46 cords of wood, at $3.50 a cord, and sold it at a profit of 25%. What was the whole gain?

17. A debt of $425.50, due in a year, was discounted at 8% for cash. What was the discount?

18. What is the premium for insuring a house and furniture, valued at $5680, at 1¼%?

19. At what price must I sell flour that cost $4.50 a barrel, to gain 20%? 25%? 33⅓%? 50%?

20. A commission merchant sells 350 bbl. of potatoes at $2.50 a barrel, and 275 bbl. of apples, at $3.25 a barrel. What is his commission for selling, at 4½%?

21. A merchant paid $5460 for a stock of goods, which he sold at an advance of 25%. After deducting $416.50 expenses, what was his gain?

22. Find the base when the percentage is 16, and the rate 4%.

23. Find the base when the percentage is 105, and the rate 35%.

24. Find the base when the percentage is $975, and the rate 15%.

25. Fifty pupils are 12½% of the whole number attending a certain school. What is the number?

26. A horse was sold at a profit of $40, which was 20% of what it cost. What did it cost?

27. Find the rate per cent when the percentage is 40, and the base 250.

28. Find the rate per cent when the percentage is $13.20 and the base $240.

29. A carriage that cost $150 was sold at a loss of 10%. How much was it sold for?

30. A grocer bought 1078 lb. of sugar, at 8½ cents a pound, and sold it at a profit of 18%. What did he gain?

31. A grocer bought a hogshead of sugar for $84.80, and sold it at 12½% profit. What was his gain?

32. A miller bought 500 bushels of wheat at $1.15 a bushel, and he sold the flour at 16⅔% advance on the cost of the wheat. What was his gain? *Ans.* $95.83⅓.

33. I bought 76 cords of wood at $3.62½ a cord, and sold it so as to gain 26%. What did I make?

34. A hatter bought 40 hats at $1.75 apiece, and sold them at a loss of 14⅔%. What was his whole loss?

35. A grocer bought 3 barrels of sugar, each containing 230 pounds, at 8¼ cents a pound, and sold it at 18$\frac{2}{11}$% profit. What was his whole gain, and what was the selling price per pound?

Ans. Gain, $10.35. Selling price per lb., 9¾ cents.

36. I bought 128 tons of coal at $5.12½ a ton, and sold it at a gain of 22%. What was the entire profit?

SIMPLE INTEREST.

190. Interest is a sum paid for the use of money.

191. Rate per cent per annum is the sum per cent paid for the use of $1.00 annually.

The rate per cent is commonly expressed decimally, as *hundredths*.

192. Legal Interest is the rate per cent established by law. It varies in different States as follows:

States and Territories.	Legal Rate of Interest.	States and Territories.	Legal Rate of Interest.
Alabama	8	Montana	10
Arizona	7	Nebraska	7
Arkansas	6	Nevada	7
California	7	New Hampshire	6
Colorado	8	New Jersey	6
Connecticut	6	New Mexico	6
Delaware	6	New York	6
Dist. of Columbia	6	North Carolina	6
Florida	8	North Dakota	7
Georgia	7	Ohio	6
Idaho	10	Oregon	8
Illinois	5	Pennsylvania	6
Indiana	6	Rhode Island	6
Iowa	6	South Carolina	7
Kansas	6	South Dakota	7
Kentucky	6	Tennessee	6
Louisiana	5	Texas	8
Maine	6	Utah	10
Maryland	6	Vermont	6
Massachusetts	6	Virginia	6
Michigan	6	Washington	10
Minnesota	7	West Virginia	6
Mississippi	6	Wisconsin	7
Missouri	6	Wyoming	12

193. Principal is the sum for the use of which interest is paid.

194. Amount is the sum of the principal and interest.

195. Simple Interest is the sum paid for the use of the principal only, during the whole time of the loan or credit.

EXAMPLES.

196. To find the interest on any sum, at any rate per cent, for years and months.

1. What is the interest on $140 for 3 years 3 months, at 7 per cent?

OPERATION.

$140	
.07	
$9.80	Int. for 1 yr.
$3\frac{1}{4}$	
245	
2940	
Ans. $31.85	Int. for 3 yr. 3 mo.

SOLUTION. — The interest on $140, for 1 yr., at 7 per cent, is .07 of the principal, or $9.80, and the interest for 3 yr. 3 mo. is $3\frac{3}{12} = 3\frac{1}{4}$ times the interest for one yr., or $3\frac{1}{4} \times \$9.80$, which is $31.85.

Rule. I. *Multiply the principal by the rate per cent, and the product will be the interest for 1 year.*

II. *Multiply this product by the time in years and fractions of a year, and the result will be the required interest.*

Find the interest and amount of:

2. $48.50, 2 yr. 6 mo. at 6%.
3. $325.41, 3 yr. 4 mo. at 5%.
4. $279.60, 1 yr. 9 mo. at 7%.
5. $26.84, 2 yr. 6 mo. at 5%.
6. $200, 1 yr. 9 mo. at 7%.
7. $750, 1 yr. 3 mo. at 5%.

197. To find the interest of $1, at 6%, for any time.

At 6%, the int. of $1 for 1 year (12 mo.) is 6 cts. $.06
For 1 mo. it will be $\frac{6}{12} = \frac{1}{2}$ ct. 005
For 2 mo., $\frac{1}{2} \times 2 = 1$ ct. 01
For 3 mo., $\frac{1}{2} \times 3 = 1\frac{1}{2}$ cts. 015
For 4 mo., $\frac{1}{2} \times 4 = 2$ cts. (and so on) 02
Again: since 30 days = 1 mo.
For 1 da. the int. is $\frac{1}{30}$ of $\frac{1}{2}$ ct. $= \frac{1}{60}$ ct. . . . . .000$\frac{1}{2}$
For 6 da., as $\frac{6}{30} = \frac{1}{5}$, it is $\frac{1}{5}$ of $\frac{1}{2}$ ct. $= \frac{1}{10}$ ct. 001
Hence we have the following rule:

Rule. *Take as many cents as equal half the number of months; one tenth of a cent for each six days, one sixtieth of a cent for each remaining day.*

1. Find the int. of $1 for 9 mo. 12 da., at 6%.

OPERATION.

$.045
.002
Ans. $.047

SOLUTION. — According to the rule the int. for 9 months is $4\frac{1}{2}$ cents, and for 12 days $\frac{2}{10}$ of a cent; the sum of these is $.047, the required int.

2. Find the int. of $1 for 17 mo. 23 da., at 6%.

OPERATION.

$.085
.003
.000$\frac{5}{6}$
Ans. $.088$\frac{5}{6}$

SOLUTION. — For 17 mo. the int. is $8\frac{1}{2}$ cts; for 18 da., $\frac{3}{10}$ of a cent; for 5 da., $\frac{5}{60}$ of a cent; their sum is $.088$\frac{5}{6}$, the required int.

3. Find the int. of $1 at 6%, for

16 mo.	*Ans.* $.08	11 mo.	*Ans.* $.055
13 mo.	$.065	2 mo. 1 da.	$.010$\frac{1}{6}$
4 mo. 18 da.	$.023	9 mo. 3 da.	$.045$\frac{1}{2}$
7 mo. 12 da.	$.037	14 mo. 4 da.	$.070$\frac{2}{3}$
10 mo. 13 da.	$.052$\frac{1}{6}$	17 mo. 27 da.	$.089$\frac{1}{2}$
5 mo. 17 da.	$.027$\frac{5}{6}$	33 mo. 20 da.	$.168$\frac{1}{3}$

Rule. *To find the interest on any sum, multiply the interest on one dollar by the given sum.*

To find the interest at any rate other than .06, find the interest at 6%, divide this interest by 6, and multiply it by the given per cent.

4. What is the interest of $450 at 6% for 10 mo. 18 da.? At 8%? At 4%? At 5%?

OPERATION.

Int. 10 mo.	$22.50
" 15 da.	1.12½
" 3 da.	22½
6)	$23.85
Int. at 1%	$ 3.97½
" " 8%	31.80
" " 4%	15.90
" " 5%	19.87½

SOLUTION. — We find the interest at 6%, as previously shown; after finding the interest at 1%, we multiply by the required rates to obtain the interest, as shown in the illustration. The interest may be found by the method shown in (**196**) or (**197**) as is most convenient in each problem to be solved.

5. What is the interest on $372 for 1 yr. 10 mo. 15 da., at 7%? *Ans.* $48.825.

6. What is the interest on $221.75 for 3 yr. 7 mo. 6 da., at 7%? *Ans.* $55.88.

7. What is the interest on $267.27 for 6 mo. 24 days, at 6%? *Ans.* $9.086.

8. What is the interest on $365 for 2 mo. 3 days, at 6%? *Ans.* $3.83.

9. What is the interest on $785.10 for 1 yr. 6 months 18 days, at 5%? *Ans.* $60.845.

10. What is the interest on $450 for 3 yr. 7 months, at 8%?

11. What is the interest on $600 for 2 yr. 8 mo., at 7%? *Ans.* $112.

12. What is the amount of $1000 for 9 mo. 15 days, at 7%? *Ans.* $1055.414.

13. What is the interest on $860 for 6 mo. 6 days, at 6%? *Ans.* $26.66.

14. What is the interest on $137.45 for 8 mo. 27 days, at 6%? *Ans.* $6.11.

15. Find the amount of $875 for 1 yr. 6 mo., at 3%. *Ans.* $914.375.

16. Find the amount of $350 for 9 mo., at 4%. *Ans.* $360.50.

17. Find the amount of $8.50 for 1 yr. 9 mo. 12 da., at 6%. *Ans.* $9.409.

18. Find the amount of $457 for 1 yr. 4 mo. 24 da., at 6%. *Ans.* $495.388.

19. Find the amount of $650 for 3 yr. 10 mo. 21 days, at 7%. *Ans.* $827.07.

20. What is the interest on $79 for 15 mo., at 7%? *Ans.* $6.912.

21. Find the amount of $86 for 5 mo., at 7%. *Ans.* $88.51.

22. What is the interest on $78.75 for 1 yr. 9 mo., at 4%? *Ans.* $5.5125.

23. What is the interest on $1750 for 30 days, at 9%? *Ans.* $13.125.

24. What is the interest on $3654.25 for 33 days, at 10%? *Ans.* $33.497.

25. Find the amount of $269.50 for 120 days, at 7%. *Ans.* $275.788.

26. Find the amount of $1625 for 1 yr. 6 mo., at 8%. *Ans.* $1820.

Find by either method the *interest* of each of the following principles, for the time specified, at 4%, 5%, 6%.

27. $48.75, 2 yr. 6 mo.
28. $276, 1 yr. 9 mo.
29. $360.40, 3 yr. 3 mo.
30. $850, 11 mo.

31. $142.50, 1 yr. 2 mo. 6 da.
32. $106.84, 2 yr. 1 mo. 15 da.
33. $76.80, 3 yr. 4 mo. 21 da.
34. $960, 2 yr. 24 da.

Find the interest and the *amount* of each of the following principles, at 6%, 7%, 8%, 10%, 5½%, 4%.

35. $327.50, 1 yr. 6 mo.
36. $1140, 2 yr. 1 mo.
37. $45.80, 4 yr. 7 mo.
38. $516.25, 2 yr. 4 mo. 3 da.
39. $1000, 7 mo. 18 da.
40. $76.84, 3 yr. 15 da.

41. Mr. Clarke paid a debt of $184.75, which had been at interest for 11 mo. 21 da., at 7%. What was the amount paid?

42. If I borrow $860 at 5½%, what will be due at the end of 9 mo. 24 da.?

43. What is due on a note of $384.50, that has run 2 yr. 8 mo. 6 da., at 6%?

44. In settling with a merchant, I give my note for $245.80. What is due in 11 mo. 15 da., at 5%?

45. Find the amount of $63.70 for 1 yr. 3 mo. 20 da., at 7%. At 8%.

46. Find the amount of $475 for 1 yr. 7 mo., at 6%.

47. Mr. Crane bought a house for $5400. He paid $2000 cash on delivery, $1500 in 9 mo., and the remainder in 1 yr. 4 mo., with 7% interest. What was the whole amount paid?

48. A man invested $1500 for 2 yr. 4 mo. 18 da., at 8%. What were the interest and the amount?

49. Mr. Johnson paid a debt of $175.50 which had been at interest 11 mo. 16 da., at 6%. What was the amount of the payment?

50. A note was given for $360, bearing interest at 10%, dated May 1st, 1891. What amount will be required to pay the note on Sept. 20, 1892?

GENERAL REVIEW.

1. Multiply the difference between 876042 and 834260 by 176. *Ans.* 7353632.

2. To 47320 add 3 times the difference between 46270 and 31032. *Ans.* 93034.

3. From 212462 + 432046, take 517240 − 230124.

4. Divide the sum of 4802 + 56010 + 20342 by 4 times the difference between 1200 and 1082. *Ans.* $171\frac{221}{288}$.

5. What is the difference between 1824624 + 15624 and 896042 − 12342? *Ans.* 956548.

6. What is the difference between 3426 × 284 and 200104? *Ans.* 772880.

7. What is the difference between 3931476 ÷ 556 and 14 × 875? *Ans.* 5179.

8. How many times can 36 be subtracted from 11772? *Ans.* 327 times.

9. How many times can 8 × 27 be taken from 1554768?

10. Divide 420 × 216 by 43756 − 42851. *Ans.* $100\frac{44}{181}$.

11. Multiply 3 times the sum of 4624 and 1036 by 2 times the difference between 375 and 296. *Ans.* 2682840.

12. What is the difference between 5 × 2.5, and 5 × 25? *Ans.* 112.5.

13. Multiply 4.05 + .025 + 1.8 by 2 − 1.875.

14. Divide 5 by .8 × .025. *Ans.* 250.

15. How many times can 1.05 be taken from 4.725? *Ans.* 4.5 times.

16. To .02 × 32.5 add 5.7 × 16.04 and from the result subtract 12.0026. *Ans.* 80.0754.

17. A farmer sold a horse for $140, a cow for $25, and 28 sheep at $2.50 a head. How much more did he receive for the horse than for the cow and sheep?

Ans. $45.

18. What is the difference between .675 ÷ .15 and .23 × .009? *Ans.* 4.49793.

19. A young lady having $75 went out shopping, and bought 14 yards of silk for a dress, at $1.50 a yard, a shawl for $15.75, a bonnet for $8, a pair of gloves for $1.12, and a pair of shoes for $1.75. How much money had she left? *Ans.* $27.38.

20. A grocer bought 12 firkins of butter, each containing 56 pounds, at 14 cents a pound; he afterwards sold 5 firkins at 16 cents, and 7 firkins at 18 cents a pound. What was his whole gain? *Ans.* $21.28.

21. A miller sold 256 barrels of flour, at $6.80 a barrel, which was $475.60 more than the wheat from which it was made cost him. What was the cost of the wheat?

Ans. $1265.20.

22. An estate worth $25640 has demands against it to the amount of $9376; after these claims are paid, the remainder is to be divided equally among 5 individuals. What will each receive? *Ans.* $3252.80.

23. If 15 tons of hay cost $311.70, how much will 1 ton cost? *Ans.* $20.78.

24. A man paid $1.24 for 15.5 pounds of beef. What was the price per pound? *Ans.* $.08.

25. A farmer exchanged 21 bushels of wheat, at $2 a bushel, for cloth worth $3 a yard. How many yards did he receive? *Ans.* 14 yards.

26. Mr. Davis who had labored for a farmer 1 year, at $15 a month, expended his whole year's wages for cows;

he paid $18 apiece for the cows. How many cows did he buy? *Ans.* 10.

27. What will be the cost of 3 hogsheads of sugar, each weighing 15 cwt., at 8 cents a pound? *Ans.* $360.

28. How many bushels of wheat, at $1.12 a bushel, can be bought for $81.76? *Ans.* 73.

29. If 140 barrels of apples cost $329, what is the cost per barrel? *Ans.* $2.35.

30. At $.825 per bushel, how many bushels of corn can be bought for $264? *Ans.* 320 bu.

31. If 25 yards of cloth can be bought for $125.25, how many yards can be bought for $751.50?

Ans. 150 yd.

32. If 150 bushels of wheat cost $435, what will 311 bushels cost? *Ans.* $901.90.

33. If 250 pounds of tea cost $135, what is the price per pound? *Ans.* $.54.

34. If 13 spoons are made from 2 lb. 10 oz. 9 pwt. of silver, what is the weight of each? *Ans.* 2 oz. 13 pwt.

35. If a man travels 20 mi. 156 rd. in a day, how far will he travel in 61 days at the same rate?

Ans. 1249 mi. 236 rd.

36. If I put 376 gal. 3 qt. 1 pt. of cider into 9 equal casks, how much do I put into each cask?

37. If a family uses $1\frac{1}{8}$ pounds of tea in 1 month, how much would it use in 1 year? *Ans.* $13\frac{1}{2}$ pounds.

38. What is the cost of 565 pounds of butter at $12\frac{1}{2}$ cents a pound? *Ans.* $70.625.

39. At $4.25 per bushel how much clover seed can be bought for $11.6875? *Ans.* 2.75 bu.

40. At $\frac{1}{16}$ of a dollar a pound, what will be the cost of 12 pounds of sugar? *Ans.* $.75.

41. At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $40\frac{4}{5}$ yards of cloth? *Ans.* $15.30.

42. How many cubic yards of earth must be dug from a cellar 40 ft. long, 30 ft. wide, 6 ft. deep; and what will be the cost of the excavation, at $.12\frac{1}{2}$ a cubic yard?

Ans. $266\frac{2}{3}$ cu. yd.; $\$33.33\frac{1}{3}$.

43. If 6 pounds of cheese cost $\$\frac{4}{5}$, how much will 10 pounds cost? *Ans.* $\$1\frac{1}{3}$.

44. How much wheat, at $1.25 a bushel, must be given for 50 bushels of corn at $.70 a bushel?

45. At 10 cents a pint, what will 189 gallons of molasses cost? *Ans.* $151.20.

46. At 40 cents a pound, what will 4 ounces of coffee cost? *Ans.* 10 cents.

47. If 3 gallons of molasses cost $\$\frac{4}{5}$, how many gallons can be bought for $4? *Ans.* $14\frac{2}{3}$ gallons.

48. At $7\frac{1}{2}$ a firkin, how many firkins of butter can be bought for $33? *Ans.* $4\frac{2}{5}$ firkins.

49. If $\frac{1}{3}$ of a yard of cloth costs $\$\frac{3}{4}$, what will one yard cost? *Ans.* $\$2\frac{1}{4}$.

50. At $3 a barrel, how many barrels of cider can be bought for $\$8\frac{3}{5}$? *Ans.* $2\frac{13}{15}$ bbl.

51. What part of 100 pounds are 16 pounds? 20 pounds? $33\frac{1}{3}$ pounds? $66\frac{2}{3}$ pounds?

52. How much wood in a load 10 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. high? *Ans.* 1 Cd. 12 cu. ft.

53. How many tons of coal may be bought for $346.125, at $9.75 per ton? *Ans.* 35.5 T.

54. What is the interest on $136.80 for 1 yr. 11 mo., at 7%? *Ans.* $18.354.

55. What will be the cost of .6 of a gallon of sirup, at $.65 a gallon? *Ans.* $.39.

56. A owns $\frac{5}{7}$ of a flouring mill, and sells $\frac{2}{5}$ of his share to B. What part of the whole has he left?

57. If 2 yards of cloth cost $\$6\frac{3}{4}$, what will 9 yards cost? *Ans.* $\$30\frac{3}{8}$.

58. What will $\frac{1}{2}$ of $\frac{5}{8}$ of a barrel of flour cost at $\$7\frac{1}{5}$ per barrel? *Ans.* $\$2\frac{1}{4}$.

59. If 1 acre of land yields 1 T. 9 cwt. 47 lb. of hay, how much will 18 acres yield?

60. How many steps, of 30 inches each, must a person take in walking 10 miles? *Ans.* 21120.

61. A speculator bought 1575 barrels of potatoes, and upon opening them, he found 15% of them spoiled; how many barrels did he lose? *Ans.* 236.25.

62. A man bought 12 bushels of chestnuts, at $\$4.50$ a bushel, and sold them at 12 cents a pint. What was his whole gain? *Ans.* $\$38.16$.

63. What is the interest on $\$300$, for 10 months 21 days, at 6%? *Ans.* $\$16.05$.

64. An agent in Chicago purchased 5450 bushels of wheat, at $\$.82$ a bushel. What was his commission at 2% on the purchase money? *Ans.* $\$89.38$.

65. A vessel loaded with 4500 bushels of corn was overtaken by a storm at sea, and it was found necessary to throw overboard 25% of the cargo; what was the whole loss, at $\$.60$ a bushel? *Ans.* $\$675$.

66. A grocer bought 2 hogsheads of molasses, at $37\frac{1}{2}$ cents a gallon, and sold it at 20% advance on the cost. What was his whole gain? *Ans.* $\$9.45$.

67. If $\frac{5}{7}$ of an acre of land is worth $\$60$, what is the value of 1 acre? *Ans.* $\$84$.

68. If an acre of land can be sown with $1\frac{1}{3}$ bushels of wheat, how many acres can be sown with 12 bushels? How many with 48 bushels?

69. If a farm is worth $\$3840$, what is $\frac{5}{8}$ of it worth? What is $\frac{3}{4}$ of it worth?

70. If a bushel of apples costs $\frac{2}{5}$ of a dollar, how many bushels may be bought for $\frac{2}{3}$ of a dollar?

71. Divide $\frac{1}{3}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $\frac{1}{3}$.

72. If 18 kegs of nails weigh 27 cwt., how much will 1 keg weigh?

73. What is the interest of $620 for 4 yr. 3 mo., at 6%? What is the amount?

74. What is the brokerage on $5462, at 4%?

75. How many pounds of butter, at 13½ cents a pound, must be given for 1230 pounds of sugar, at 8 cents a pound? *Ans.* 728⅛ pounds.

76. If 168 bu. 1 pk. 6 qt. of corn are equally divided among 35 men, how much will each man receive?

Ans. 4 bu. 3 pk. 2 qt.

77. What will be the cost of lathing and plastering overhead, a room 36 feet long and 27 feet wide, at 28 cents a square yard? *Ans.* $30.24.

78. How much land, at $2.50 an acre, must be given in exchange for 360 acres, at $3.75 an acre?

79. What is the amount of $564.58, for 3 yr. 5 mo. 12 da., at 6%? *Ans.* $681.448.

80. How much sugar, at 9 cents a pound, should be given for 6½ cwt. of tobacco, at 40 cents a pound?

81. How many times may a jug which holds ⅛ of a gallon, be filled from a cask containing 128 gallons?

82. A man having $25000, invested 30% of it in bonds and mortgages, 45% of it in bank stocks, and the remainder in railroad stock. How much did he invest in railroad stock? *Ans.* $6250.

83. How many times can a box holding 4 bu. 3 pk. 2 qt., be filled from 336 bu. 3 pk. 4 qt.? *Ans.* 70 times.

84. How many cords of wood are there in 17 piles, each 11 feet long, 4 feet wide, and 6 feet high?

85. If the price of 1 acre of land is $32¾, what is the value of ⅞ of an acre? *Ans.* $28 21/32.

86. If 36.48 yards of cloth cost $54.72, what will 14.25 yards cost? *Ans.* $21.375.

87. What number of times will a wheel 14 ft. 10 in. in circumference, turn round in traveling 11 mi. 255 rd. 12 ft. 6 in.? *Ans.* 4200 times.

88. A man bought a farm of 136 acres, at $94 an acre; he paid $475 for fencing and the improvements, and then sold it for 14% advance on the whole cost. What was his whole gain? *Ans.* $1856.26.

89. If $13.342 will pay for 17.5 bushels of barley, how many bushels can be bought for $76.24?

Ans. 100 bushels.

90. A lady having $40.50, spent 40% of it for dry goods. What had she left? *Ans.* $24.30.

91. A man bought a house and lot for $6425; in the course of five years it increased in value 110%. What was the property then worth? *Ans.* $13492.50.

92. What will a broker charge to change $560 uncurrent money for current money, at 3%? *Ans.* $16.80.

93. If 4 hogsheads of oil cost $181.44, what is the cost of 1 pint? *Ans.* 9 cents.

94. What will 5 casks of rice cost, each weighing 165 pounds, at $5\frac{1}{2}$ cents a pound? *Ans.* $45.37\frac{1}{2}$.

95. At the battle of Waterloo, which took place June 18, 1815, the estimated loss of the French was 40000 men; of the Prussians, 38000; of the Belgians, 8000; of the Hanoverians, 3500; and of the English, 12000. What was the entire loss of life in this battle?

96. A merchant gave his note for $5200. He paid at one time $2500, and at another $175. What remained due? *Ans.* $2525.

97. A traveler who was 1300 miles from home, traveled homeward 235 miles in one week, in the next week 275 miles, in the next 325 miles, and in the next 280 miles. How far had he still to go before he would reach home? *Ans.* 185 mi.

98. If one mile of railroad requires 116 tons of iron worth $65 a ton, what will be the cost of sufficient iron to construct a road 128 miles in length? *Ans.* $965120.

99. If a young man receives a salary of $500 a year, and pays $240 for board, $125 for clothing, $75 for books, and $50 for other expenses, how much will he have left at the end of the year? *Ans.* $10.

100. A farmer sold 184 bushels of wheat, at $2 a bushel, for which he received 67 yards of cloth, at $4 a yard, and the rest in groceries. What did his groceries cost him?

101. The first of three numbers is 4, the second is 8 times the first, and the third is 9 times the second. What is their sum? *Ans.* 324.

102. A man purchased a house for 2375 dollars, and expended 340 dollars in repairs; he then sold it for railroad stock worth 867 dollars, and 235 acres of western land valued at 8 dollars an acre. What did he gain by the trade? *Ans.* 32 dollars.

103. If 256 is multiplied by 25, the product diminished by 625, and the remainder divided by 35, what will be the quotient? *Ans.* 165.

104. Two men start from different places, distant 189 miles, and travel towards each other; one goes 4 miles, and the other 5 miles an hour. In how many hours will they meet?

105. A farmer raised 1575 bushels of wheat, and 900 bushels of corn. He sold 807 bushels of wheat, and 391 bushels of corn to A, and the remainder to B. How much of each did he sell to B?

Ans. 768 bushels of wheat, and 509 of corn.

106. How many men are there in an army consisting of 52714 infantry, 5110 cavalry, 6250 dragoons, 3927 light-horse, 928 artillery, 250 sappers, and 406 miners?

107. John has $8546, which is $342 less than 4 times as much as Charles has. How many dollars has Charles? *Ans.* $2222.

108. The product of two numbers is 31383450, and one of the numbers is 4050. What is the other number?

109. A horse and a wagon cost $270; the horse cost $1\frac{1}{4}$ times as much as the wagon. What was the cost of the wagon?

110. Four children inherited $2250 each; but one dying, the remaining three inherited the whole. What was the share of each? *Ans.* $3000.

111. A tailor made 3 suits of clothes; for the first suit he used $2\frac{1}{8}$ yards of broadcloth, $3\frac{1}{16}$ yards of cassimere, and $\frac{7}{8}$ of a yard of satin; for the second suit 2.25 yards of broadcloth, 2.875 yards of cassimere, and 1 yard of satin; and for the third suit $5\frac{1}{16}$ yards of broadcloth, and $1\frac{1}{8}$ yards of satin. How many yards of each kind of goods did he use? How many yards of all?

Ans. to last, 18.375.

112. A man sold three houses; for the first he received $2475, for the second $840 less than he received for the first, and for the third as much as for the other two. What did he receive for the three? *Ans.* $8220.

113. Two men travel in opposite directions, one at the rate of 35 miles a day, and the other at the rate of 40 miles a day. How far apart are they at the end of 6 days?

114. Two men travel in the same direction, one at the rate of 35 miles a day, and the other at the rate of 40 miles a day. How far apart are they at the end of 6 days?

115. If the sum of two fractions is $\frac{5}{8}$, and one of them is $\frac{9}{20}$, what is the other? *Ans.* $\frac{7}{40}$.

116. If the smaller of two fractions is $\frac{24}{81}$, and their difference is $\frac{7}{9 \cdot 8}$, what is the greater?

ADDITIONAL TABLES.

198. The old **French Linear, and Land Measure,** is still used to some extent in Louisiana, and in other French settlements in the United States.

TABLE.

12 Lines = 1 Inch. 6 Feet = 1 Toise.
12 Inches = 1 Foot. 32 Toises = 1 Arpent.
900 Square Toises = 1 Square Arpent.

The *French Foot* equals nearly 12.8 American inches.

The *Arpent* is the old French name for *Acre*, and contains nearly $\frac{4}{5}$ of an English acre.

In Texas, New Mexico, and in other Spanish settlements of the United States, the following denominations are still used:

TABLE.

1000000 Square Varas = 1 Labor = 177.136 Acres (American).
25 Labors = 1 League = 4428.4 Acres "

The *Spanish Foot* = 11.11+ in. (Am.); 1 Vara = 33⅓ in. (Am.); 100 Varas = 100 Yards, and 1900.8 Varas = 1 Mile.

OTHER DENOMINATIONS IN USE.

5000 Varas Square = 1 Square League.
1000 Varas Square = 1 Labor, or $\frac{1}{25}$ League.
5645.376 Square Varas = 4840 Square Yards = 1 Acre.
23.76 Square Varas = 1 Square Chain = $\frac{1}{10}$ Acre.
1900.8 Varas Square = 1 Section = 640 Acres.

199. The following table on page 224 will assist farmers in making an accurate estimate of the amount of land in different fields under cultivation:

TABLE.

10 rods	×	16 rods	= 1 A.	220 feet	×	198 feet	= 1 A.
8 "	×	20 "	= 1 "	110 "	×	396 "	= 1 "
5 "	×	32 "	= 1 "	60 "	×	726 "	= 1 "
4 "	×	40 "	= 1 "	120 "	×	363 "	= 1 "
5 yds.	×	968 yds.	= 1 "	400 "	×	108.9 "	= 1 "
10 "	×	484 "	= 1 "	200 "	×	217.8 "	= 1 "
20 "	×	242 "	= 1 "	100 "	×	435.6 "	= 1 "
40 "	×	121 "	= 1 "				

200. The following table will often be found convenient for taking *inside* dimensions:

A box 24 in. × 24 in. × 14.7 in. will contain *a barrel* of 31½ gallons.
A box 15 in. × 14 in. × 11 in. will contain 10 *gallons.*
A box 8¼ in. × 7 in. × 4 in. will contain *a gallon.*
A box 4 in. × 4 in. × 3.6 in. will contain *a quart.*
A box 24 in. × 28 in. × 16 in. will contain 5 *bushels.*
A box 16 in. × 12 in. × 11.2 in. will contain *a bushel.*
A box 12 in. × 11.2 in. × 8 in. will contain *a half-bushel.*
A box 7 in. × 6.4 in. × 12 in. will contain *a peck.*
A box 8.4 in. × 8 in. × 4 in. will contain *a half-peck* or 4 dry quarts.
A box 6 in. by 5¾ in., and 4 in. deep, will contain *a half-gallon.*
A box 4 in. by 4 in., and 2 1/10 in. deep, will contain *a pint.*

201. Nails are put up 100 pounds to the keg.

Size.	Length, inches.	Nails in a lb.	Size.	Length, inches.	Nails in a lb.	Size.	Length, inches.	Nails in a lb.
3*d* fine blued.	1¼	725	30*d* com. blued.	4½	16	6*d* casing.	2	210
3*d* com. "	1¼	400	40*d* " "	5	14	8*d* "	2½	134
4*d* " "	1½	300	50*d* " "	5½	11	10*d* "	3	78
6*d* " "	2	150	60*d* " "	6	8	6*d* finishing.	2	317
8*d* " "	2½	85	6*d* fence.	2	80	8*d* "	2½	208
10*d* " "	3	60	8*d* "	2½	50	16*d* "	3	126
12*d* " "	3¼	50	10*d* "	3	30	6*d* clinching.	2	118
16*d* " "	3½	40	12*d* "	3¼	27	8*d* "	2½	80
20*d* " "	4	20	16*d* "	3½	20	10*d* "	3	45

5 lb. of 4*d* or 3¾ lb. of 3*d* will put on 1000 shingles.
5¾ lb. of 3*d* fine will put on 1000 lath.